GRAVITATIONAL LENSING OF THE COSMIC MICROWAVE BACKGROUND

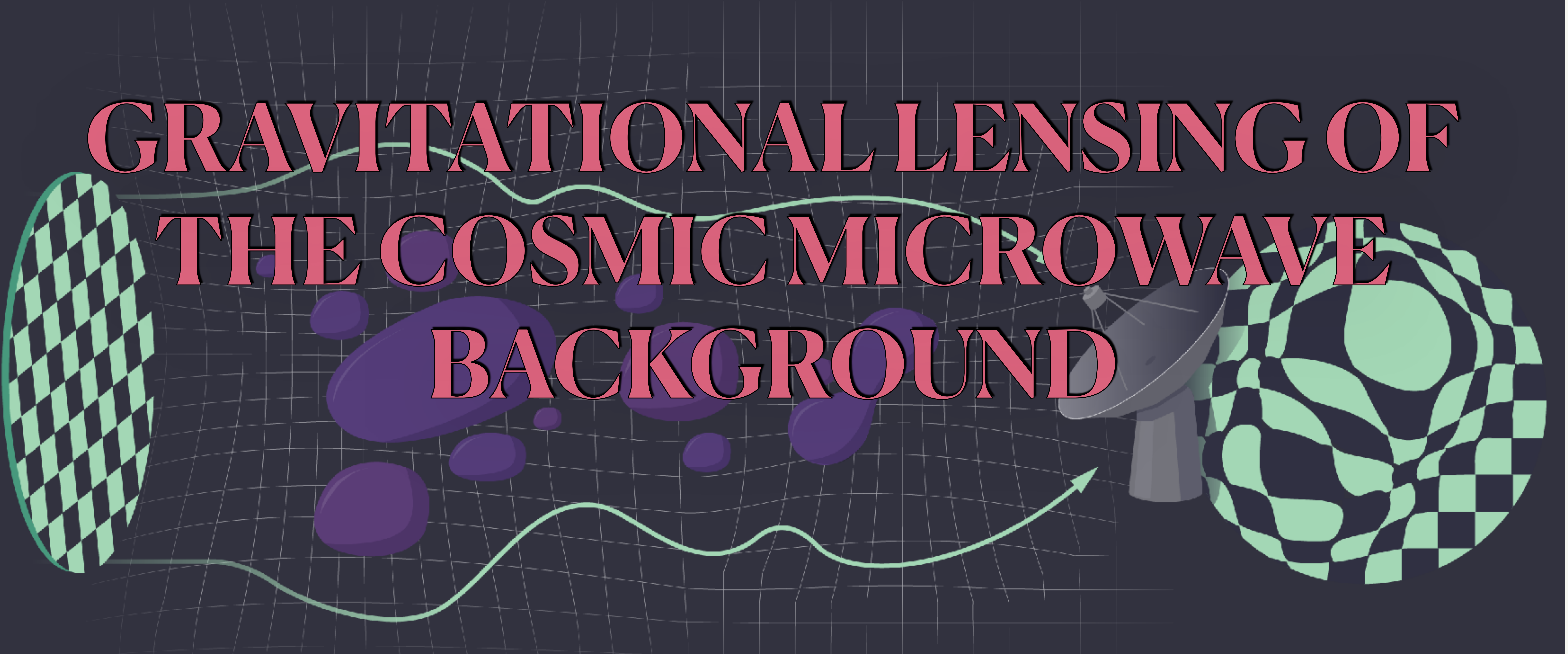

Image credit: Lucy Reading-Ikkanda/Simons Foundation

Irene Abril Cabezas, 2026 Flatiron CMB Summer School

Outline

1. Motivation
2. CMB lensing basics
3. Effect on the CMB
4. Delensing
5. Lensing reconstruction
6. Lensing power spectrum measurement and challenges

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DISCLAIMER: None of the ideas my own. This is what I have learned from many, including Blake Sherwin, (Hanson &) Lewis & Challinor, **Frank Qu**, Sigurd Naess, Jo Dunkley, Hu & Okamoto, Mat Madhavacheril, Alex van Engelen, Toshiya Namikawa

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Why study CMB lensing?

“Observable that has emerged as a robust and powerful probe for Cosmology”

<u>As a nuisance:</u>

- It has a small but non-negligible effect on the **CMB power spectra**
- Lensing effects generate a curl-like (B-mode) polarisation pattern that acts as confusion noise for CMB polarisation experiments targeting a **primordial gravitational wave signal**

<u>Just for being itself:</u>

- It probes the large-scale matter distribution in the Universe:
  - How do cosmic structures growth? (σ_8)
  - What are the properties/masses of neutrinos? ($\sum m_\nu$)

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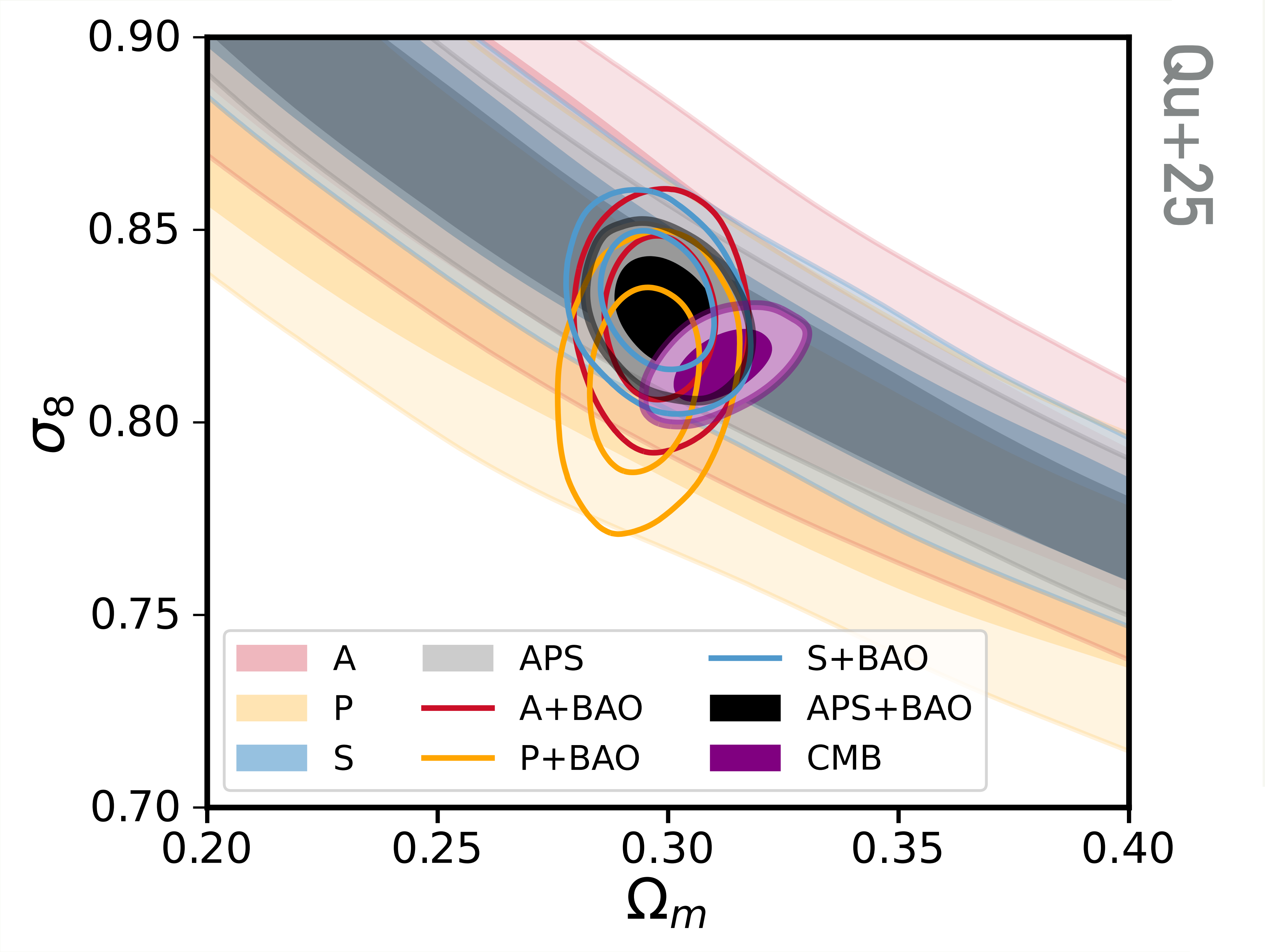
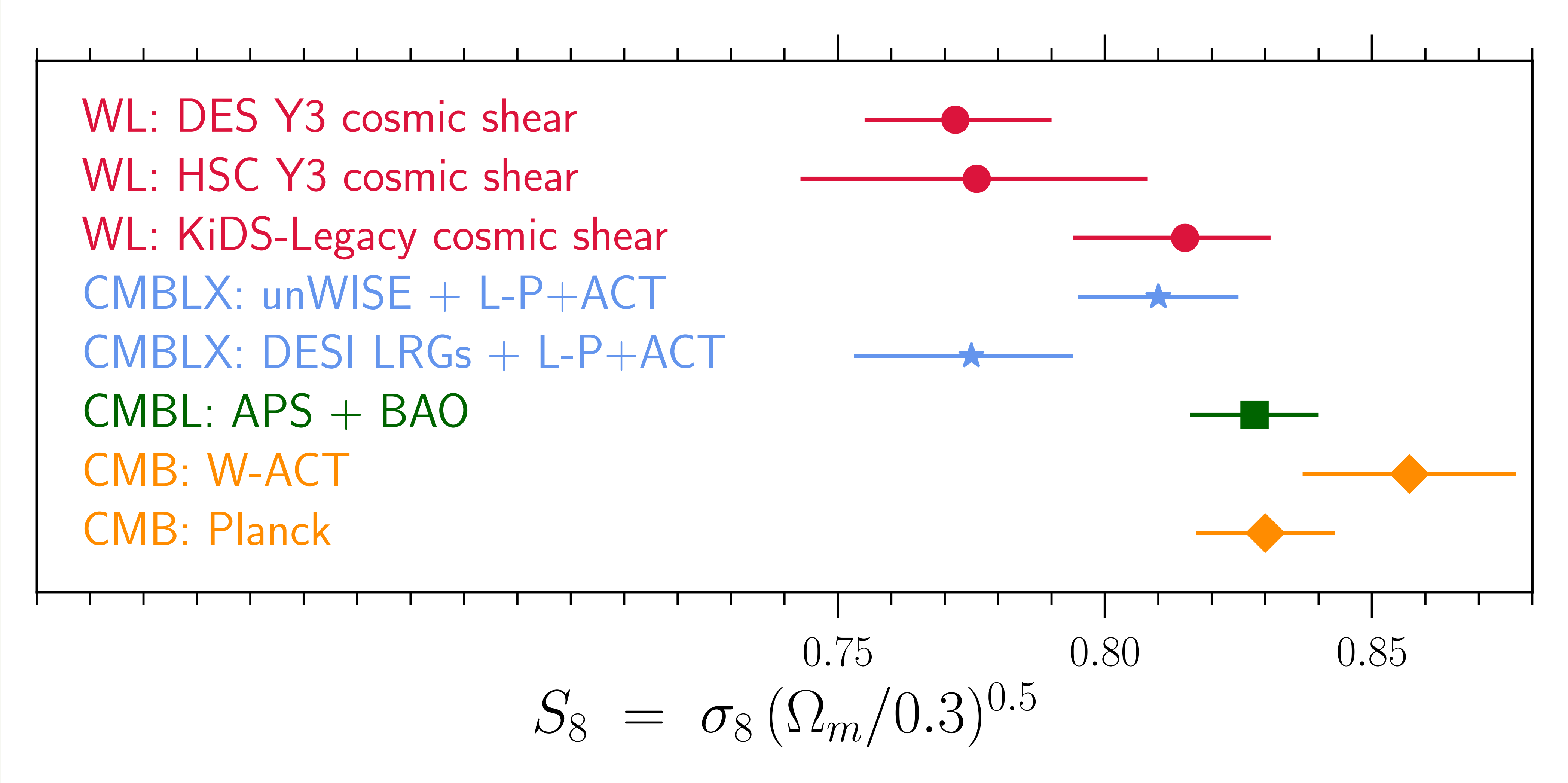
Structure growth over time as a powerful probe of new physics





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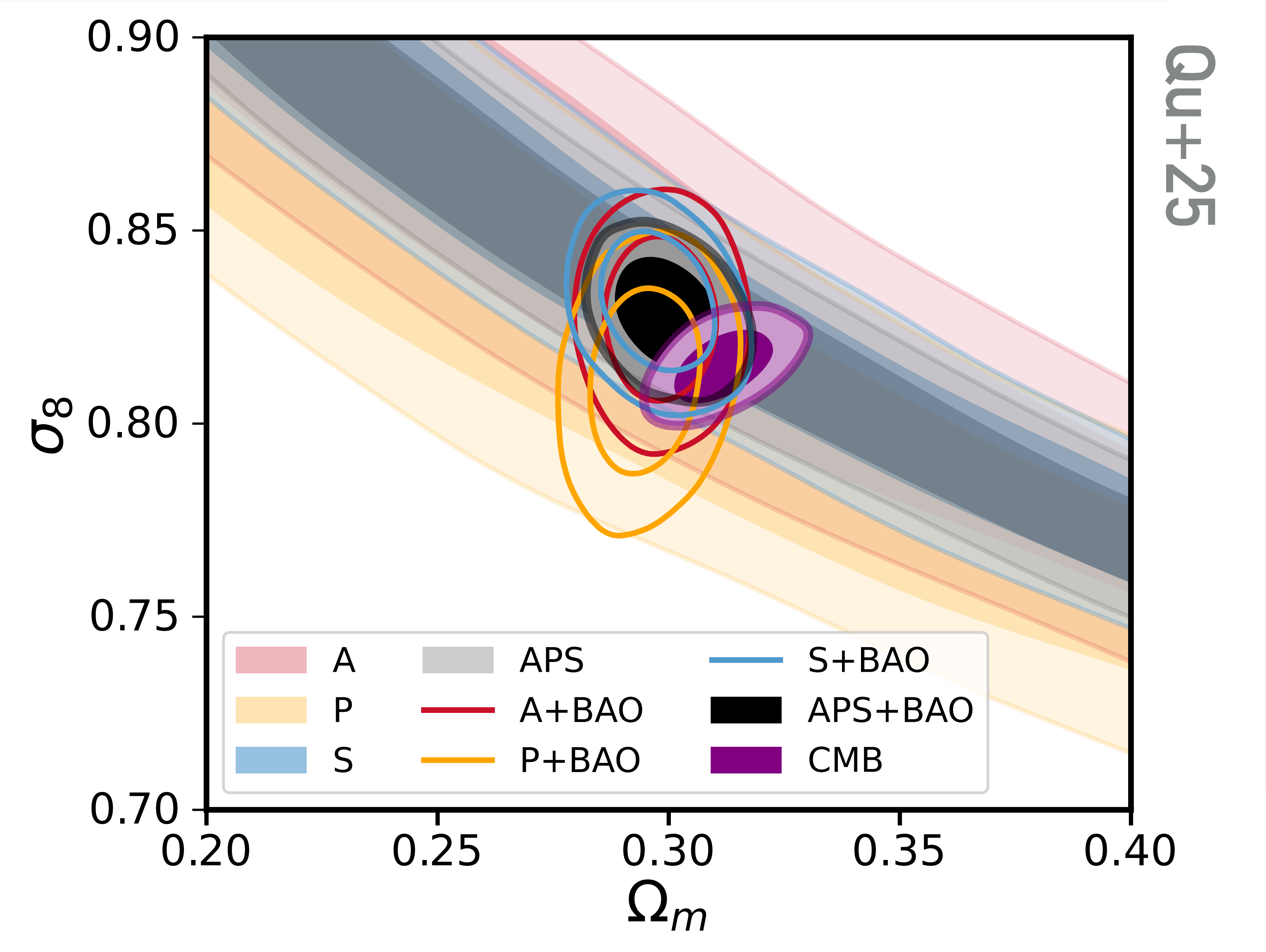

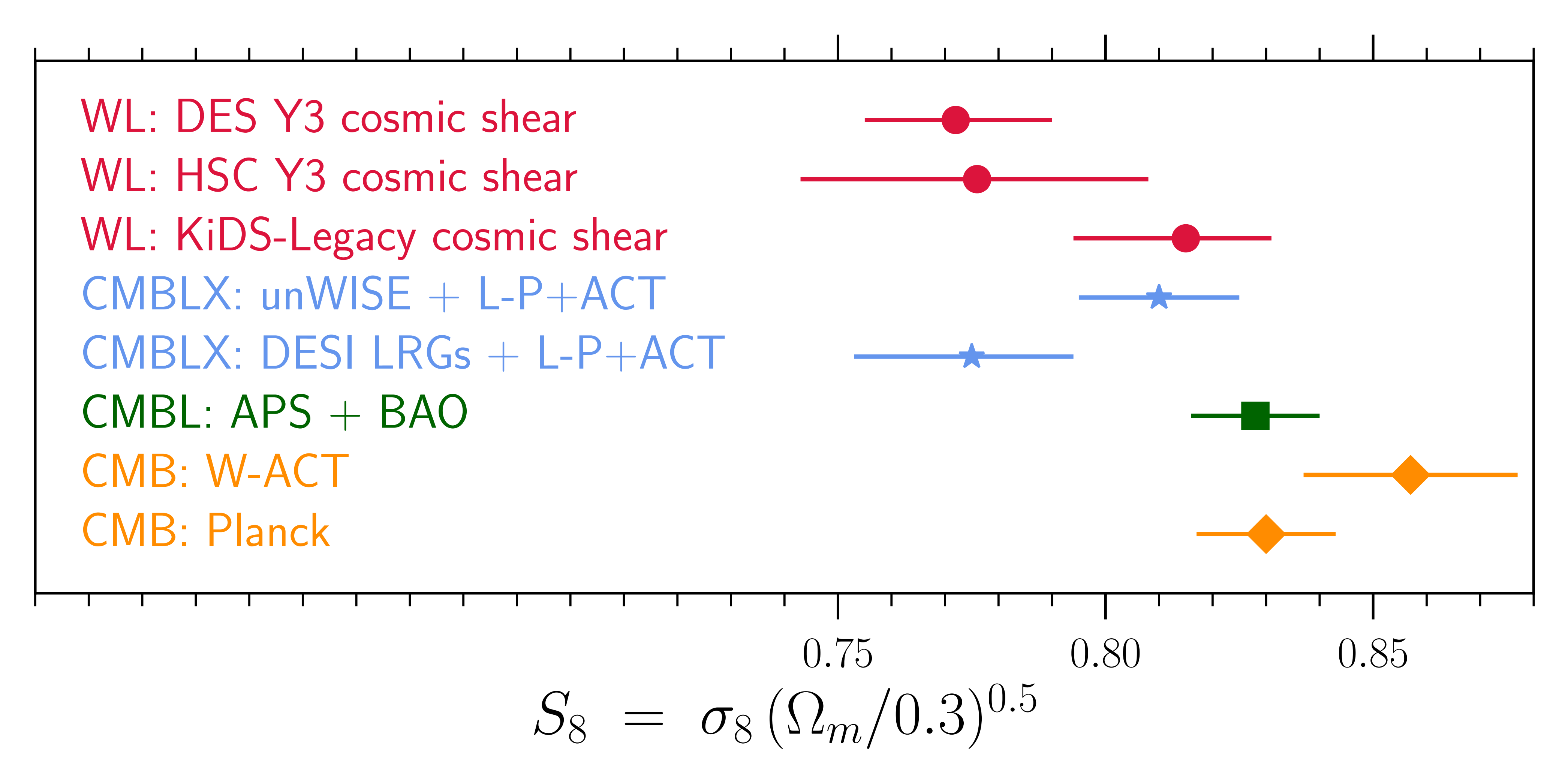

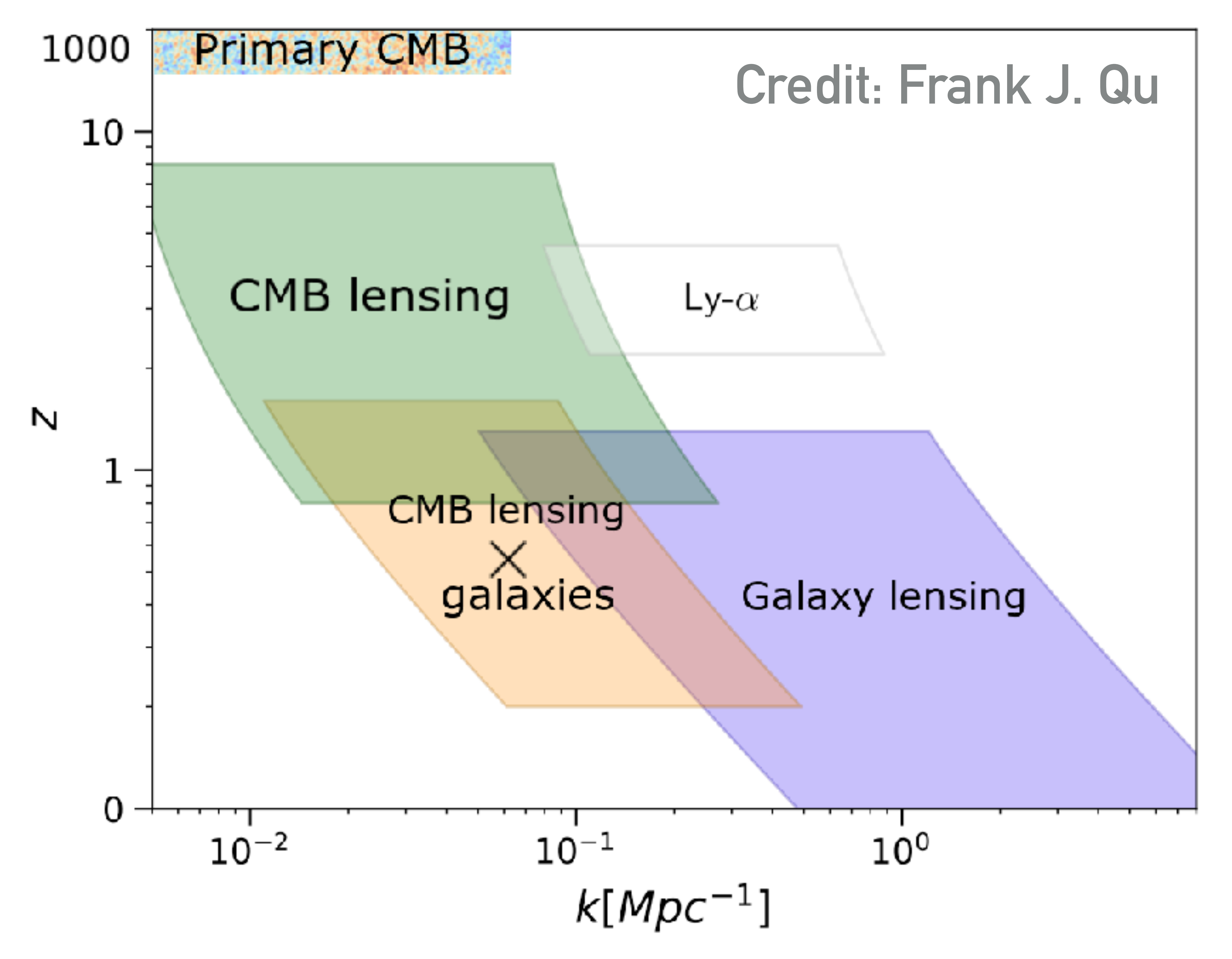

CMB lensing
order of magnitude + geometry

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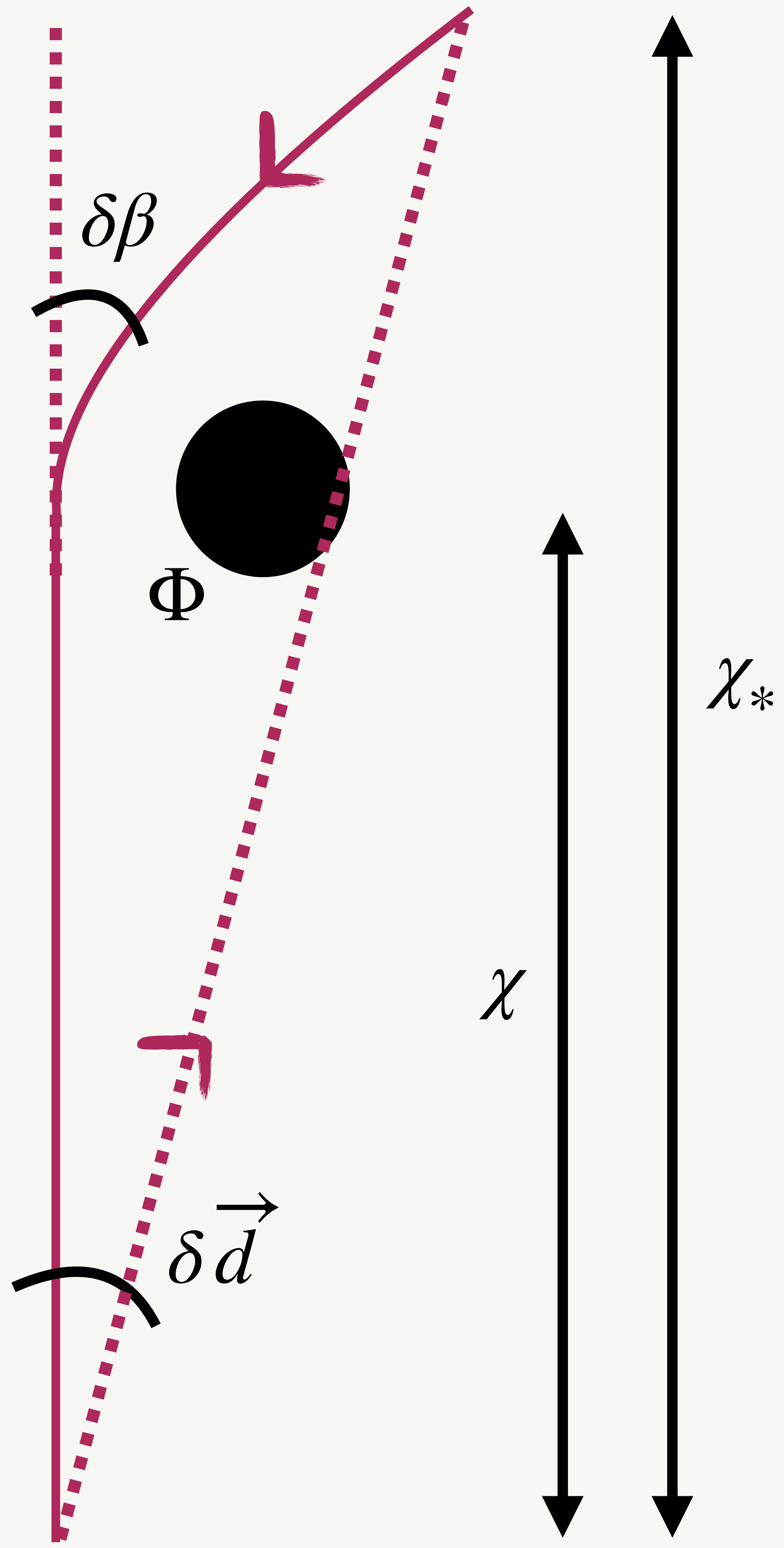
$$\delta\beta = -2\delta\chi\nabla_{\perp}\Phi \quad \Longrightarrow \quad \delta\vec{d} = -2\left(\frac{\chi_* - \chi}{\chi_*}\right)\delta\chi\nabla_{\perp}\Phi$$

Figure credit: Anthony Challinor



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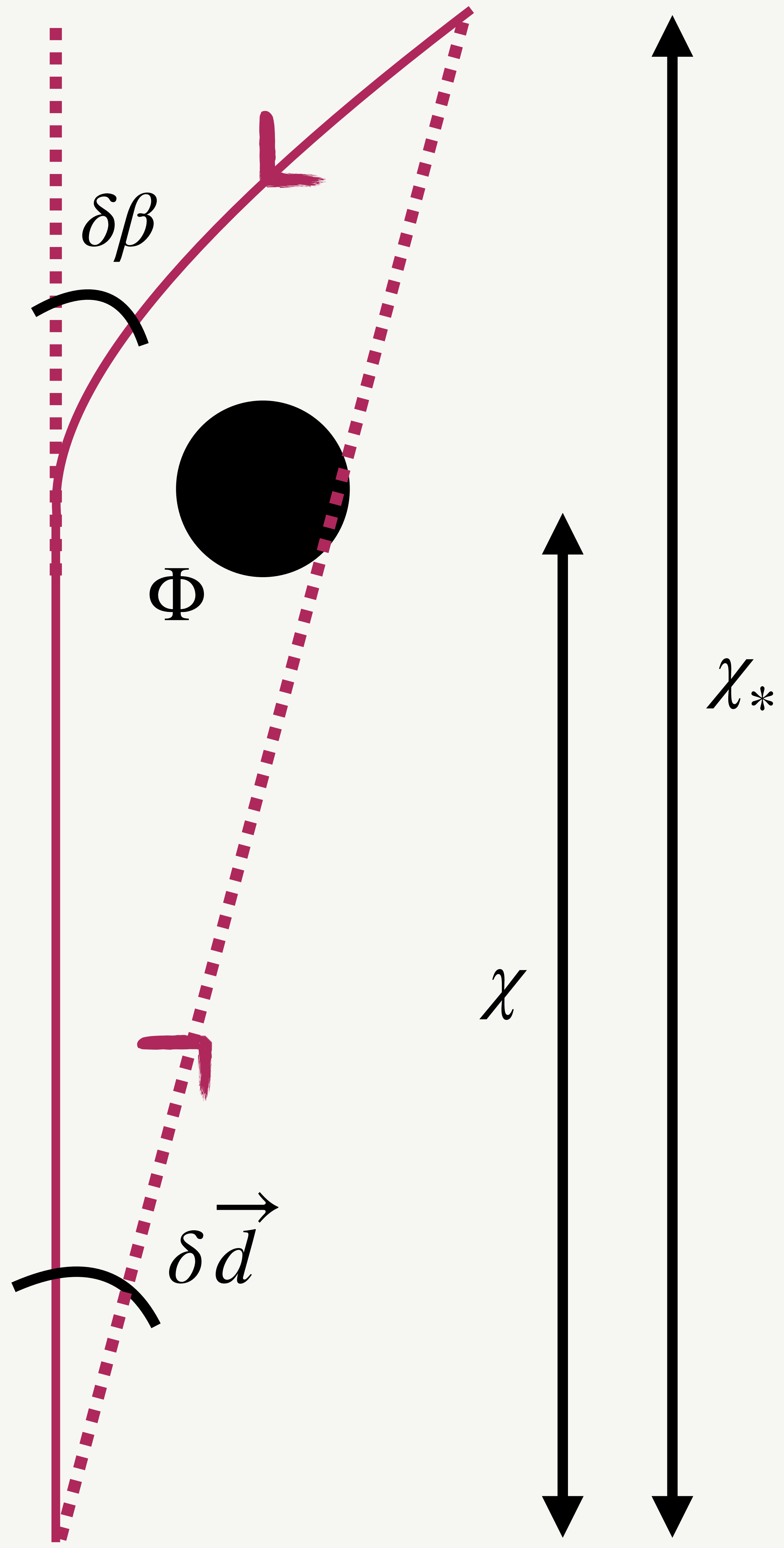
- Summing up all small deflections along photon's path:

$$\vec{d}\,(\hat{n}) = -2\int_{0}^{\chi_{*}} \mathrm{d}\chi \left(\frac{\chi_{*}-\chi}{\chi_{*}}\right)\nabla_{\perp}\Phi$$



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And other ways to present equivalent information $\left(\vec{d}, \phi, \kappa\right)$

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The lensing potential is a projection of the 3D gravitational potentials in our Universe (with some distance weighting):

$$\phi\left(\hat{n}\right) = -2 \int_0^{\chi_*} \mathrm{d}\chi \left(\frac{\chi_* - \chi}{\chi \chi_*}\right) \Phi$$

Relationship between angular gradient and the actual 3D perpendicular gradient: $\nabla = \partial_\theta = \chi \nabla_\perp$

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How do we probe the projected total mass density in each direction?

$$\kappa \equiv -\frac{1}{2} \nabla^2 \phi \simeq \int_0^{\chi_*} \mathrm{d}\chi \frac{\chi\left(\chi_* - \chi\right)}{\chi_*} \nabla^2_{3D} \Phi = \Omega_{m,0} \int_0^{\chi_*} \mathrm{d}\chi \tilde{W}\left(\chi\right) \delta\left(\hat{n}, \chi\right)$$

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Poisson's equation:

$$\nabla^2_{\mathrm{3D}} \Phi = \frac{3}{2a} \Omega_{m,0} H_0^2 \delta$$

Geometric projection kernel:

$$\tilde{W}\left(\chi\right) = \frac{3H_0^2}{2a} \frac{\chi\left(\chi^* - \chi\right)}{\chi^*}$$

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κ is the quantity that directly probes the mass distribution in our Universe

fractional mass over-density

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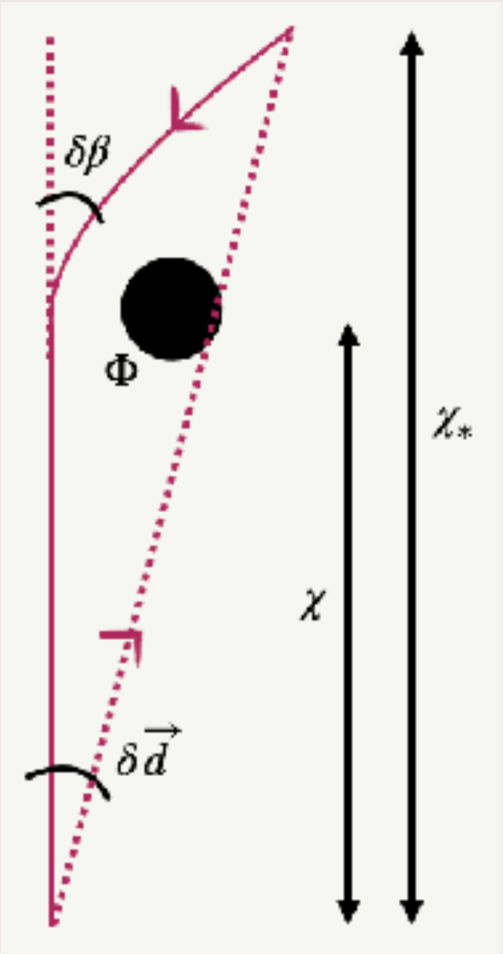
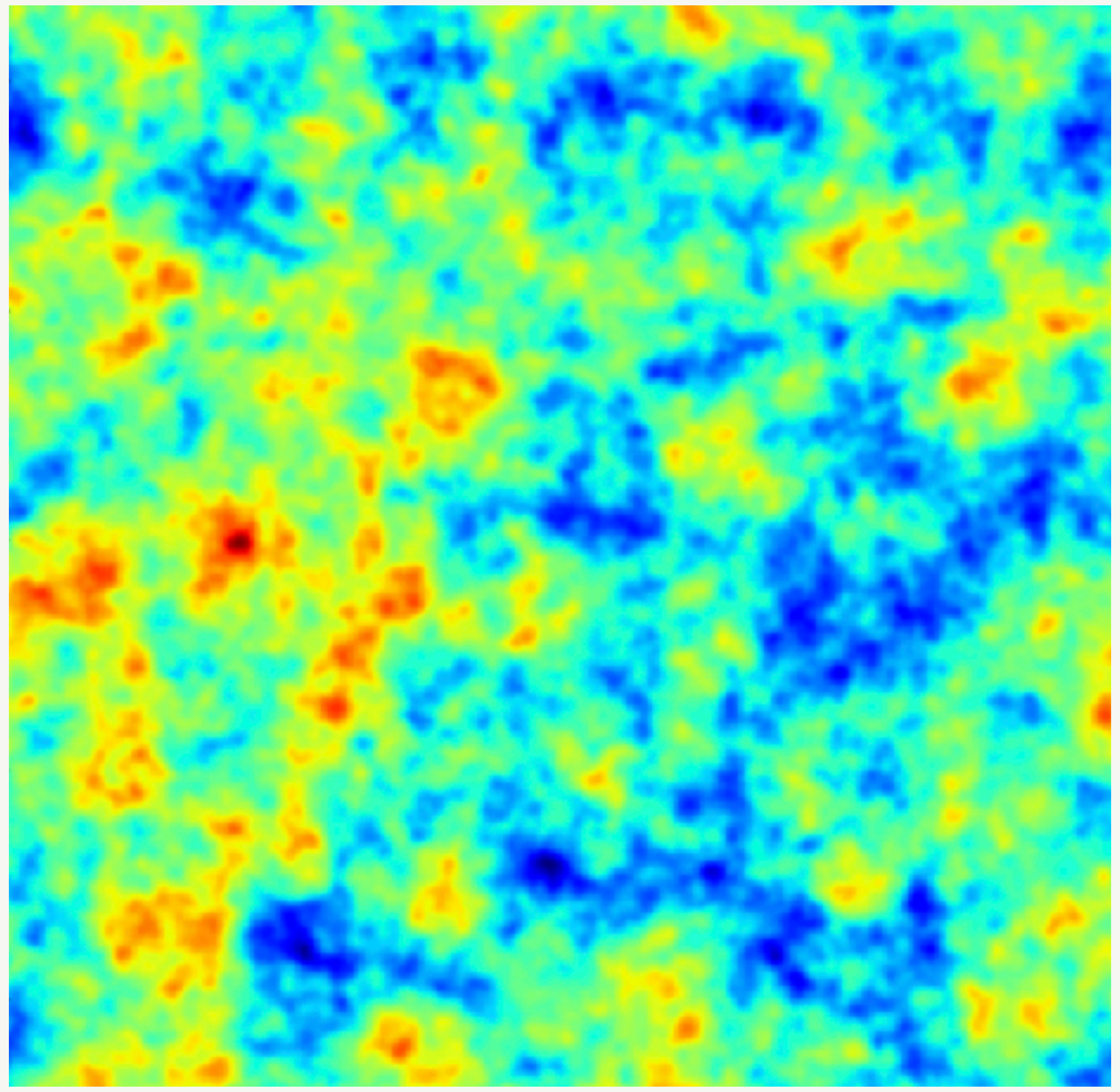
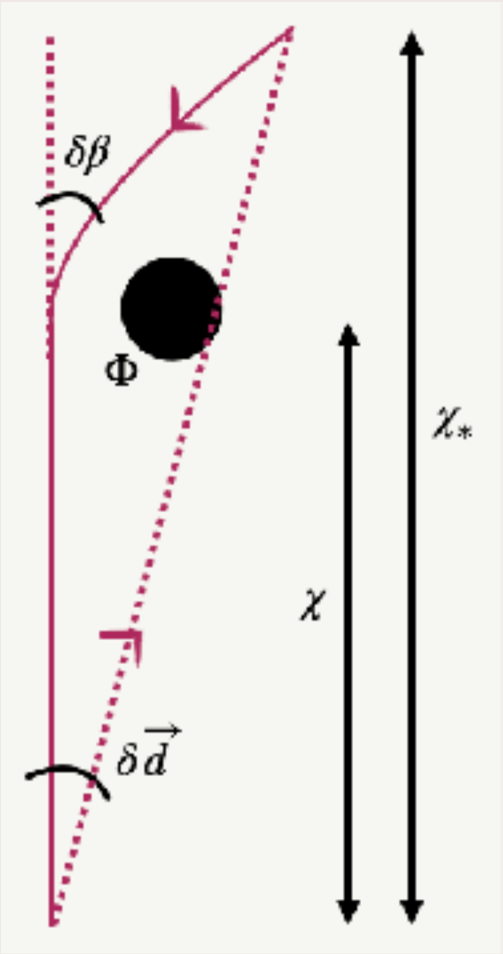


The lensed CMB temperature in a direction $\hat{n}$, $\widetilde{T}\left(\hat{n}\right)$, is given by the unlensed temperature in the deflected direction:

$$\widetilde{T}\left(\hat{n}\right) = T\left(\hat{n}'\right) = T\left(\hat{n} + \vec{d}\right), \quad \vec{d} = \nabla\phi$$

c.f. similar expressions can be written for the polarisation fields Q, U





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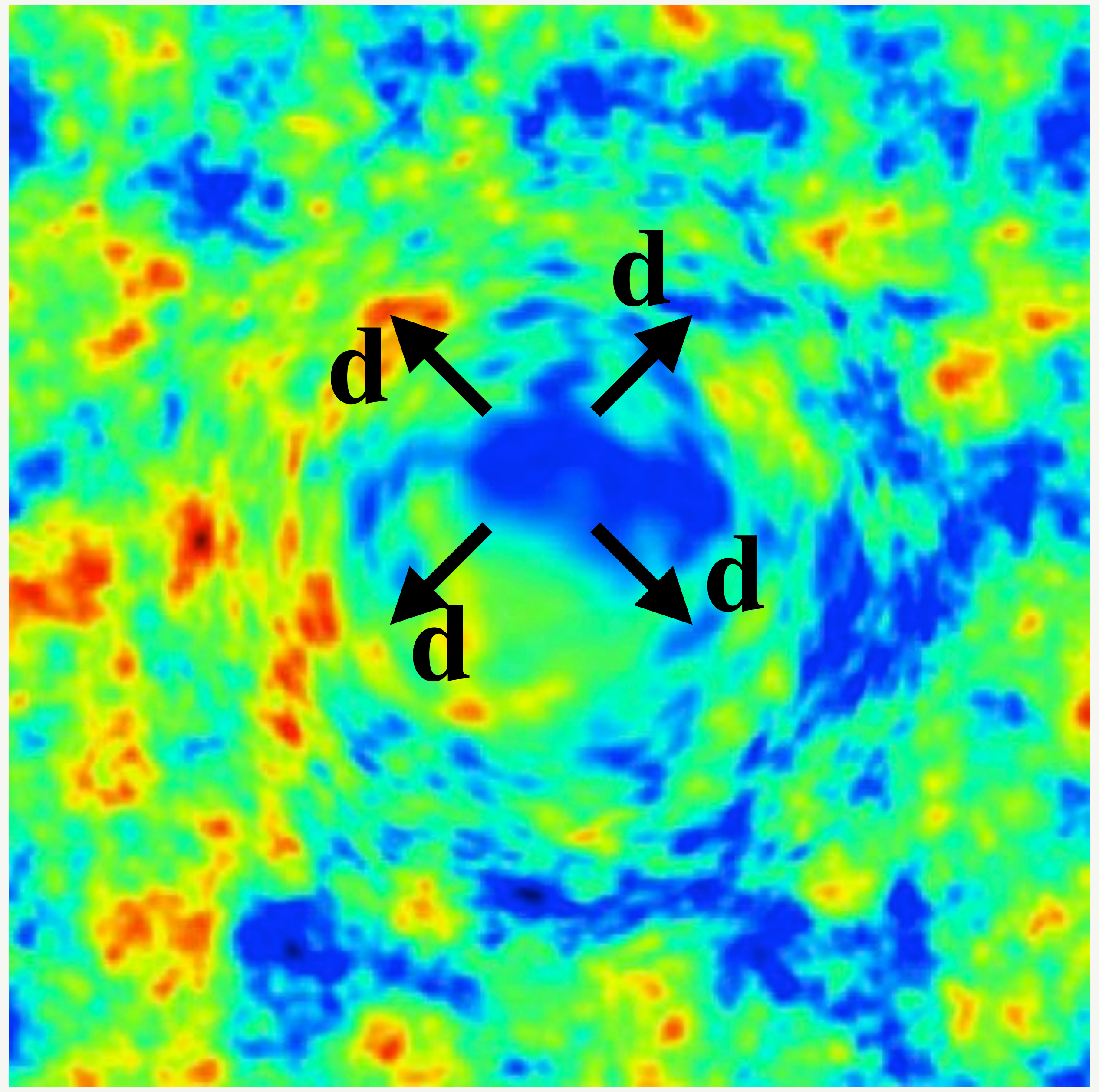

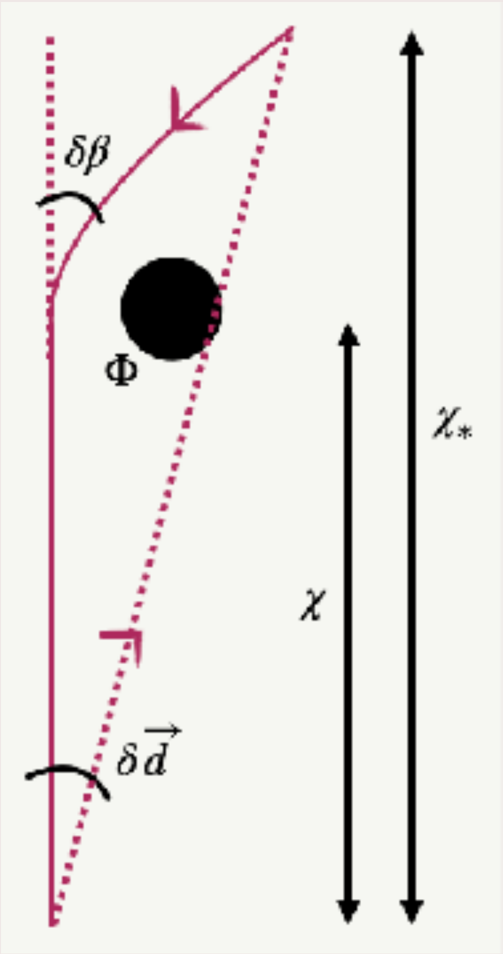

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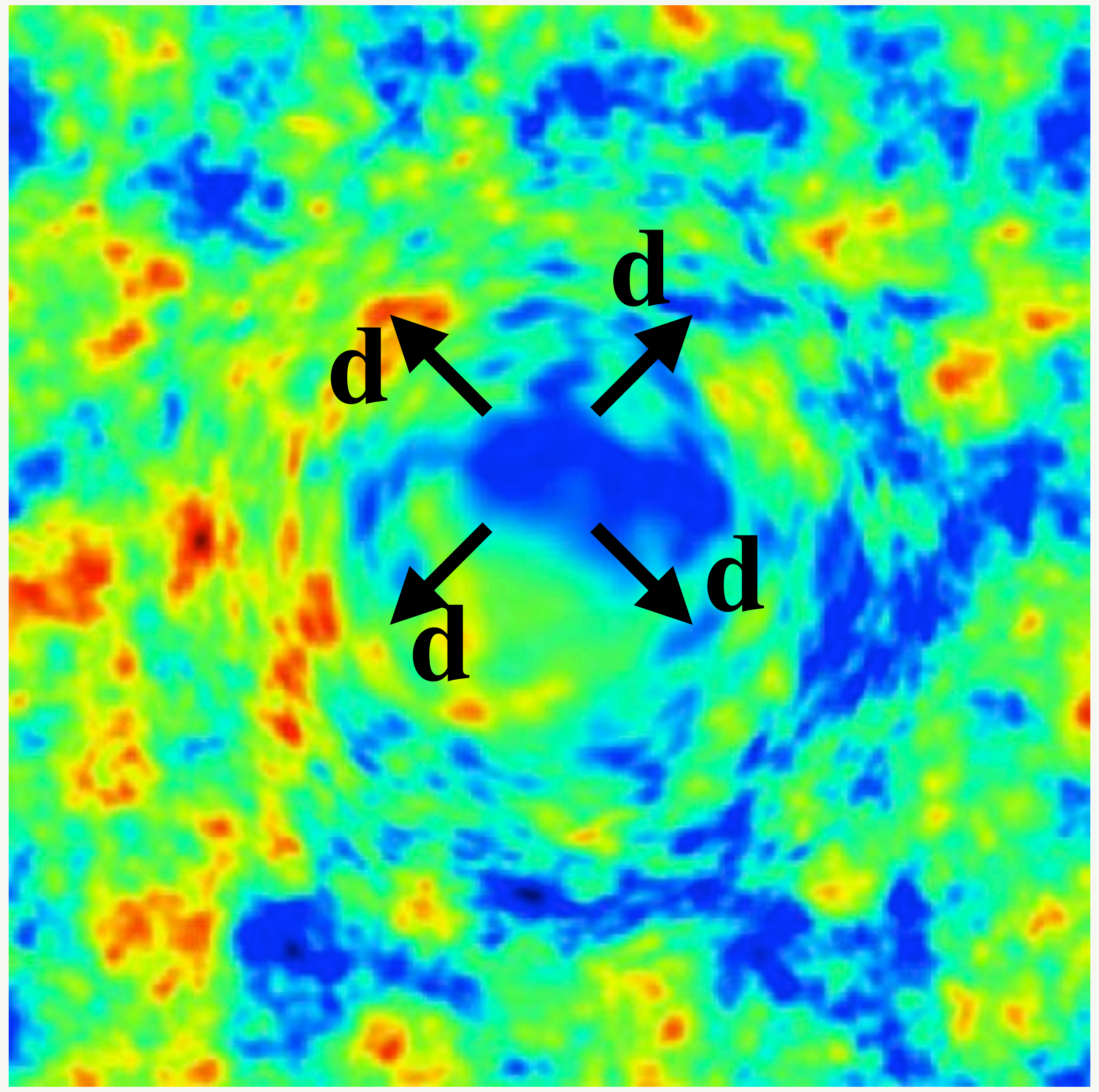
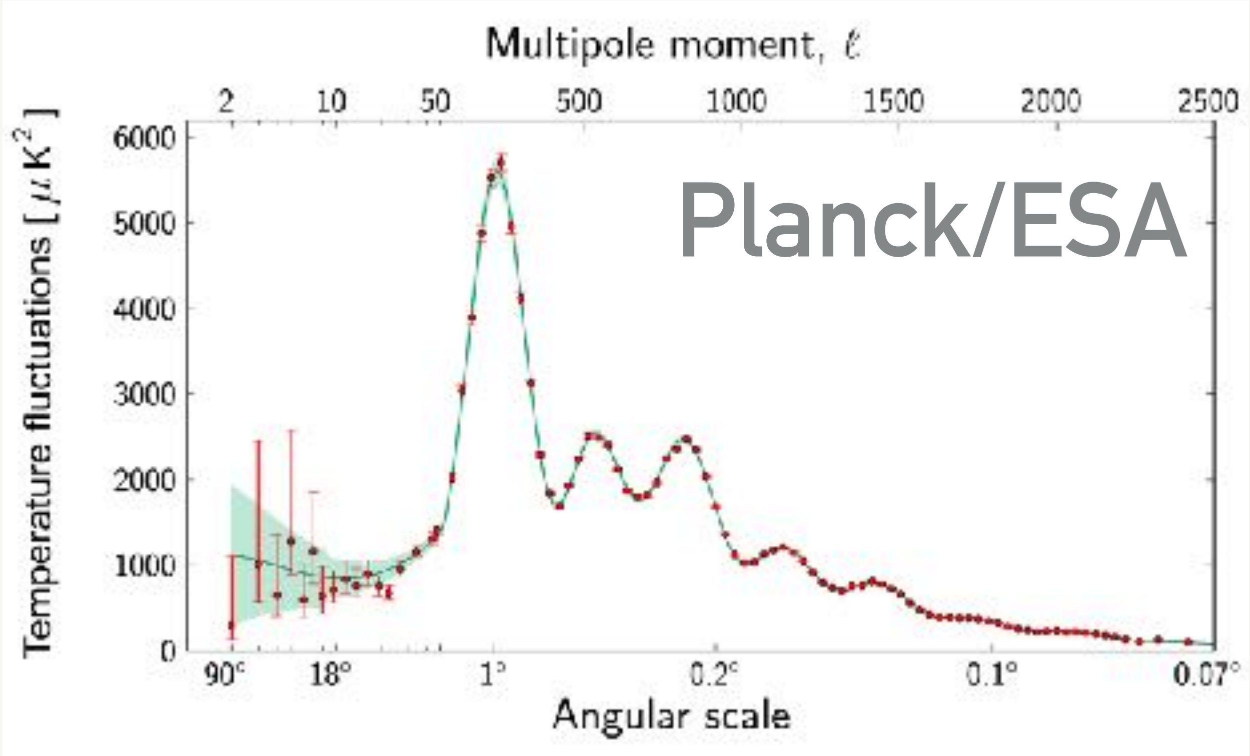
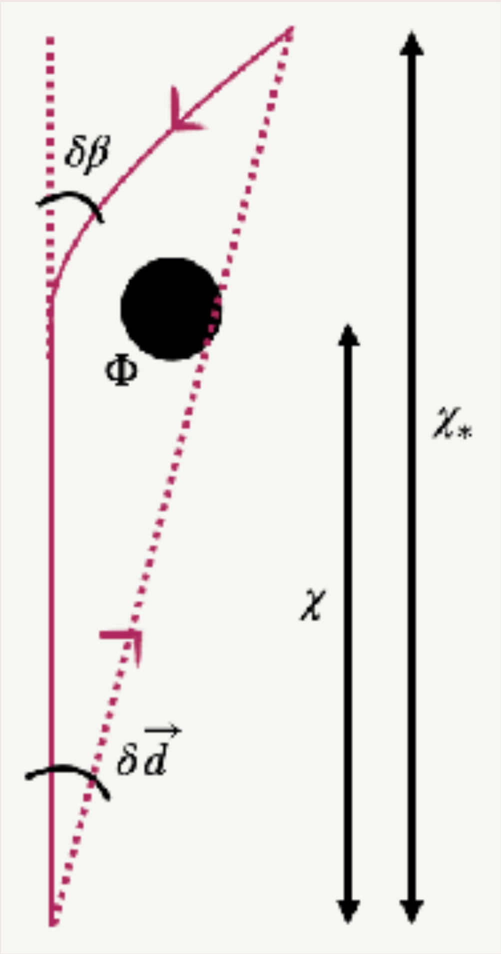


Lensing effects on the CMB power spectra





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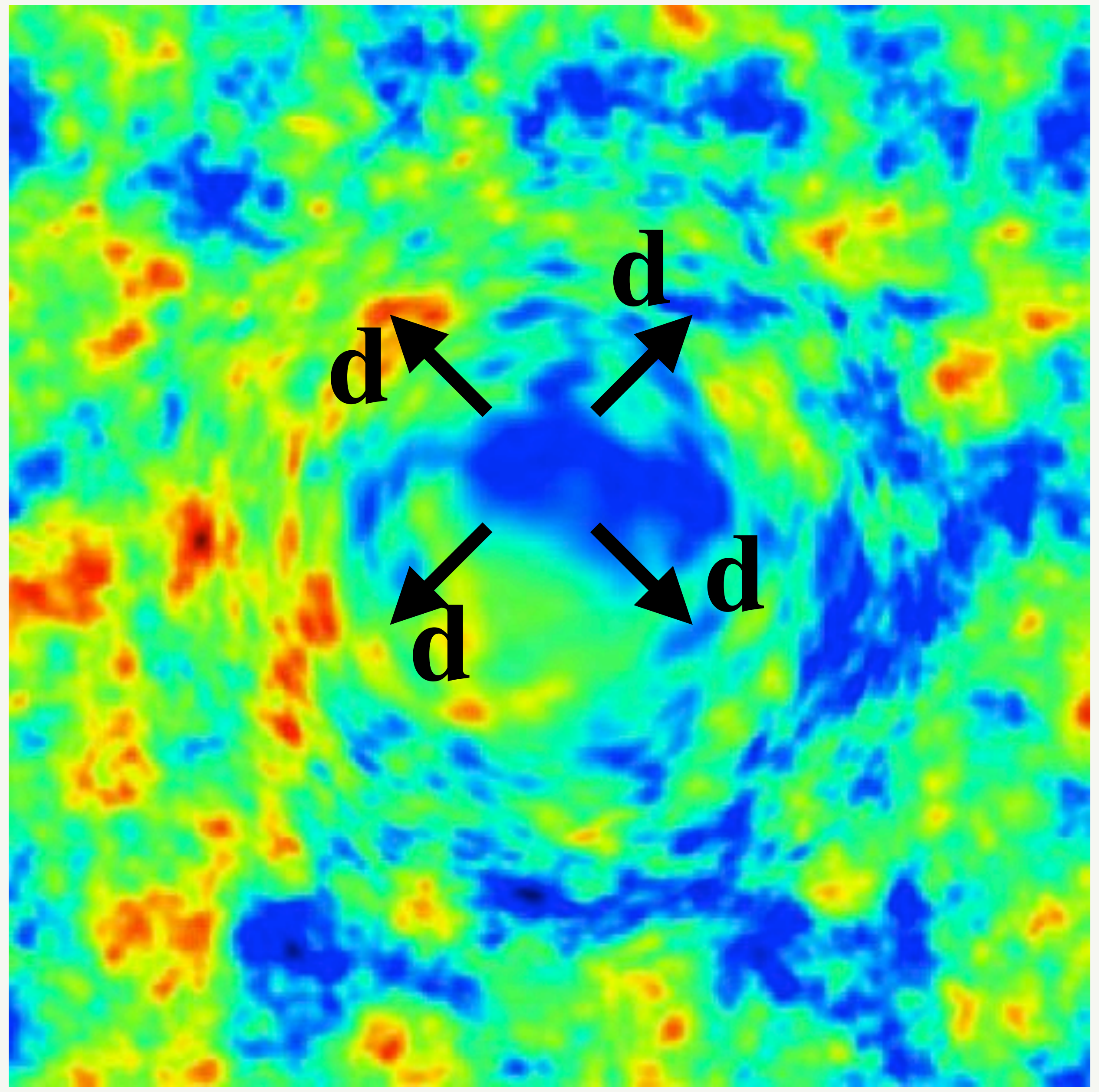

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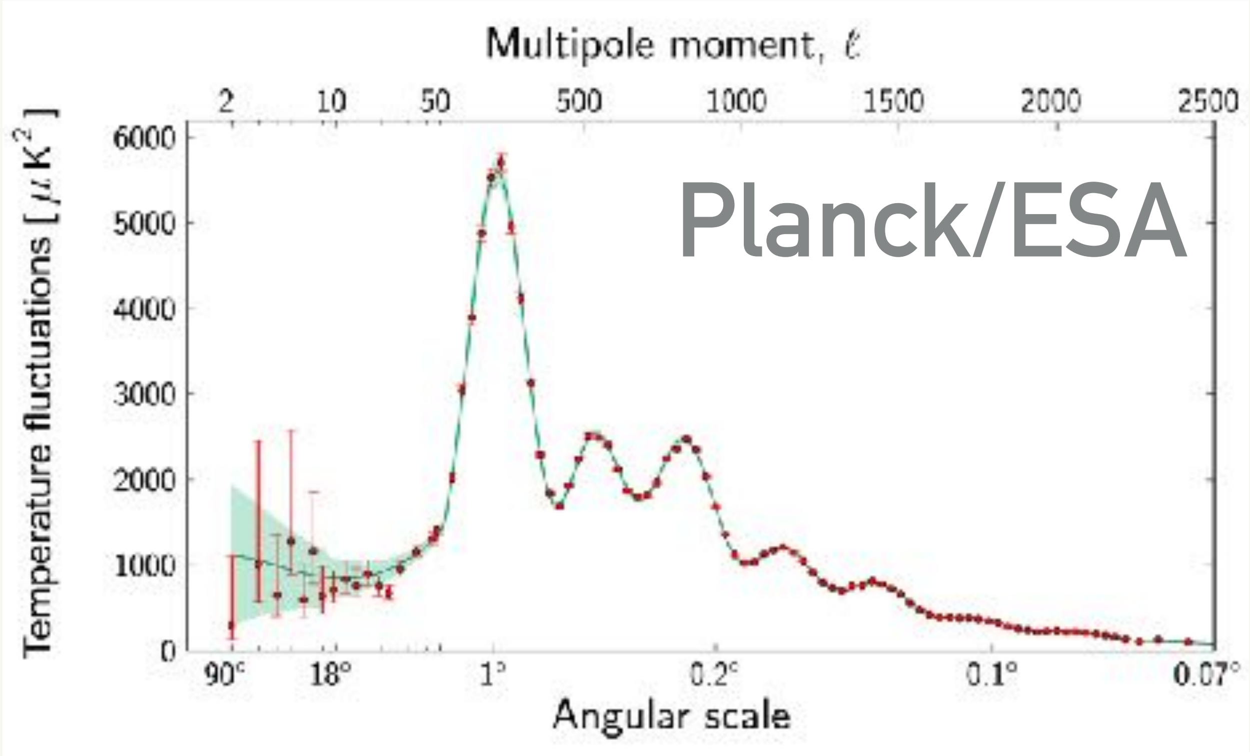

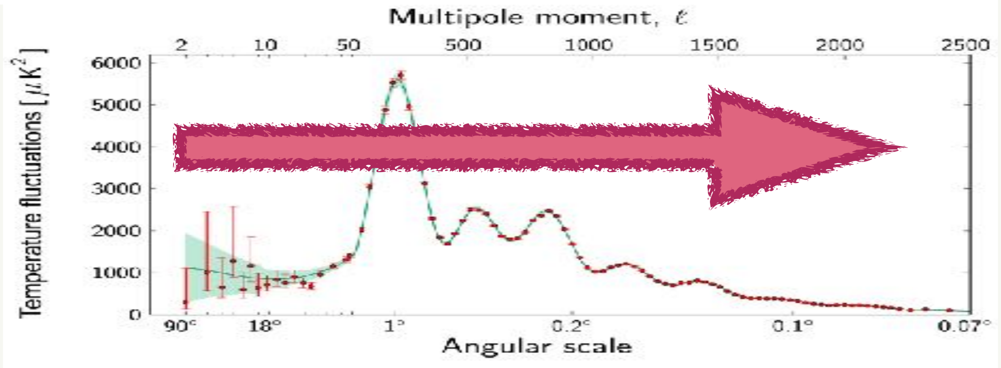

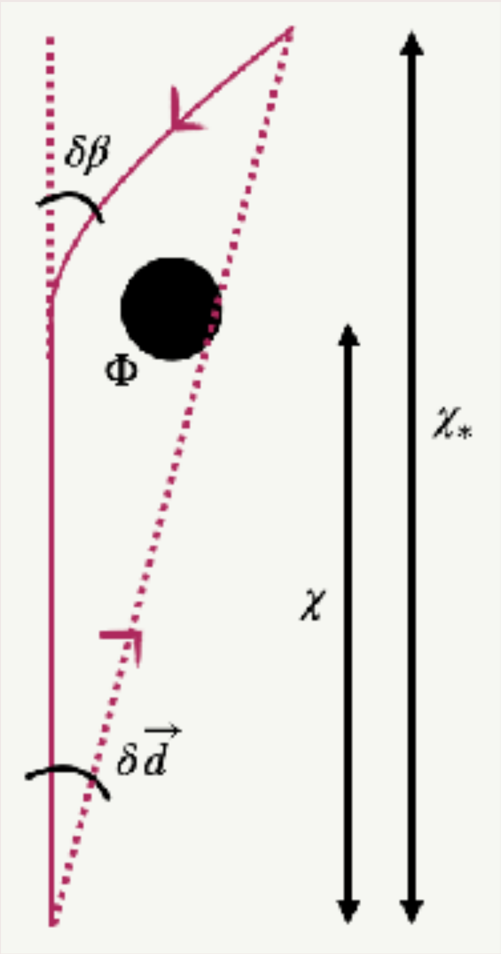

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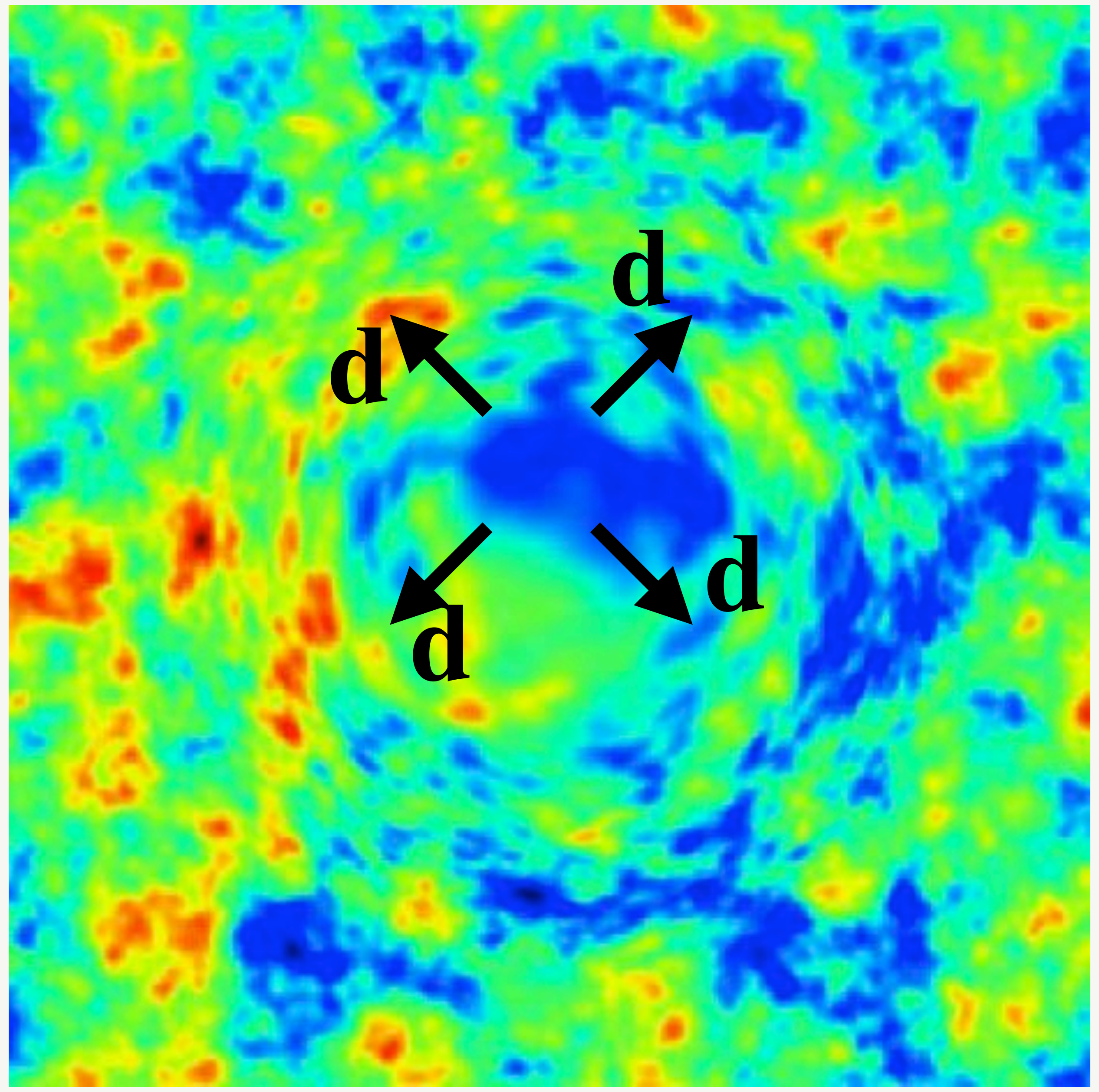

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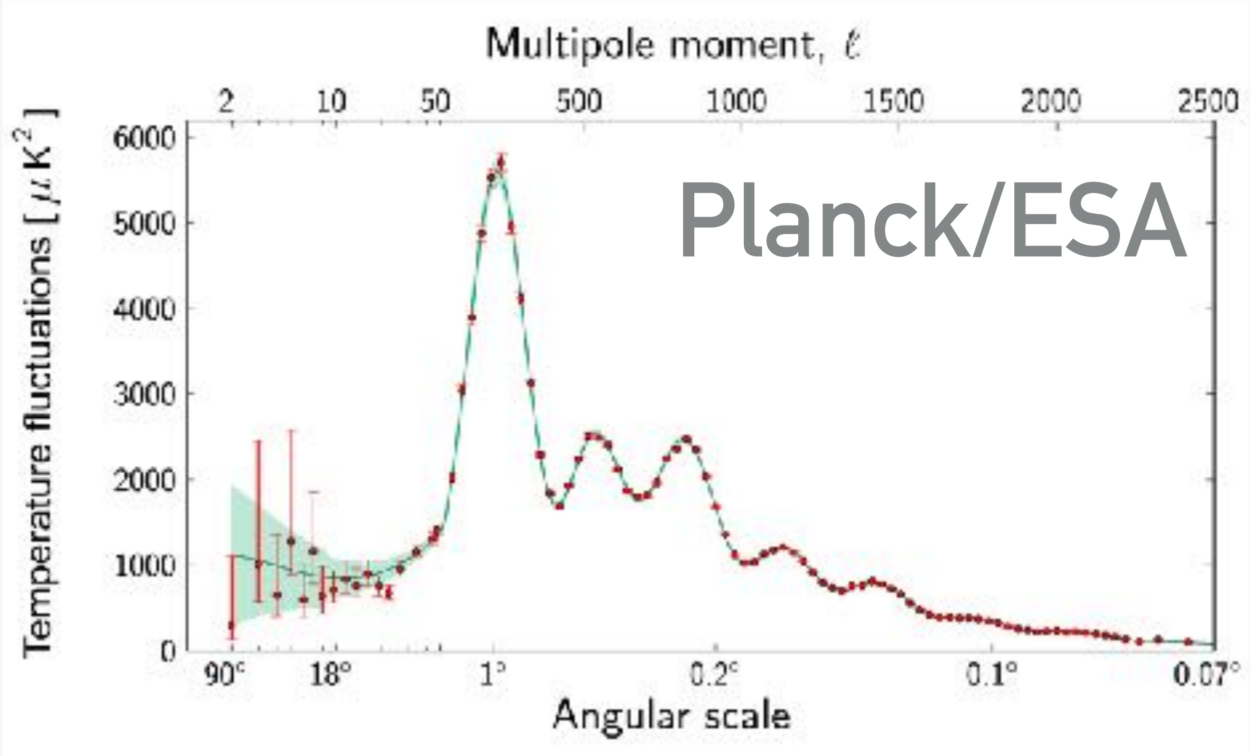

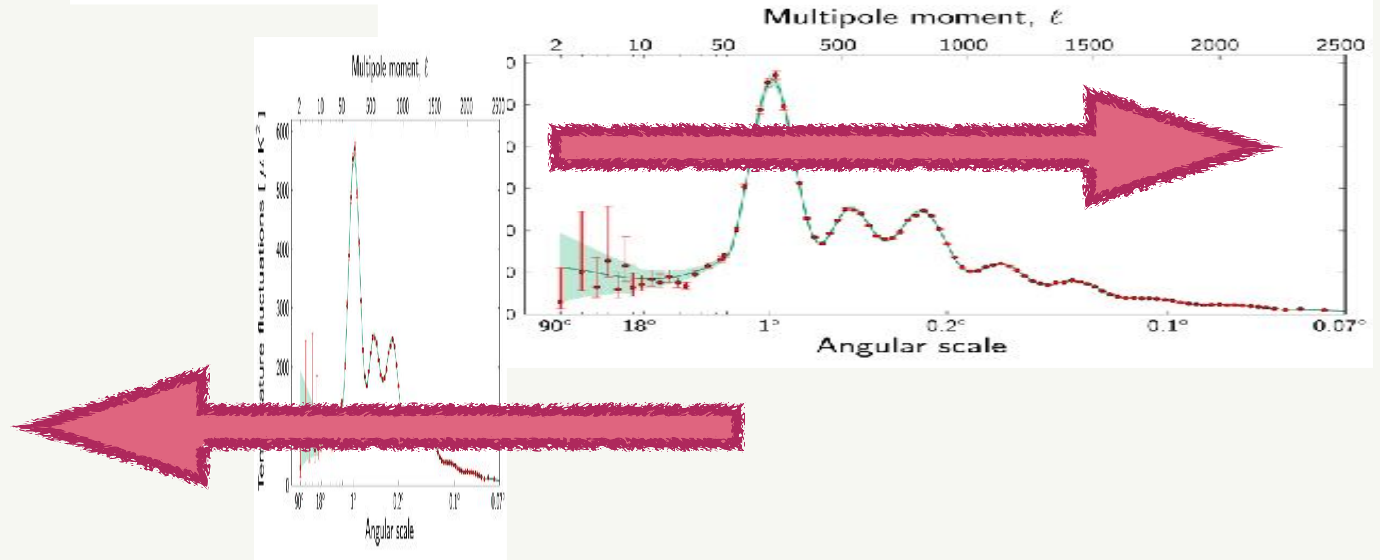

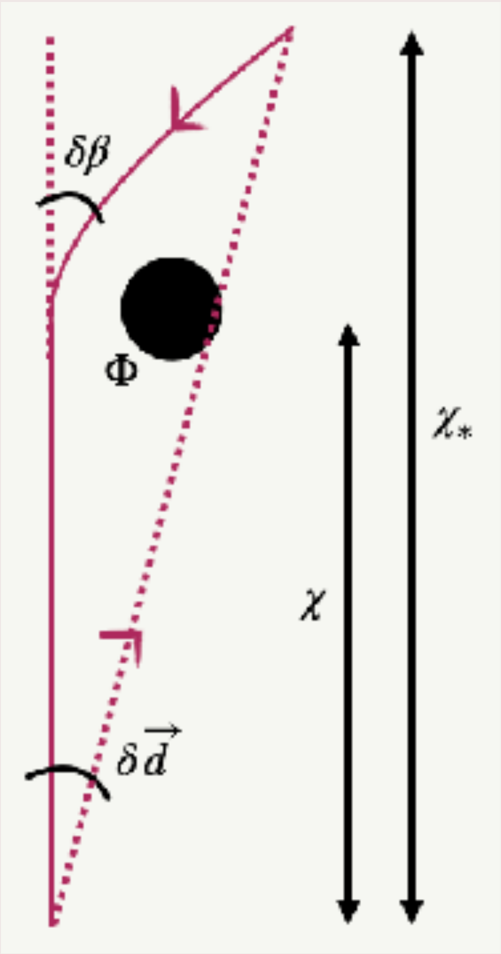

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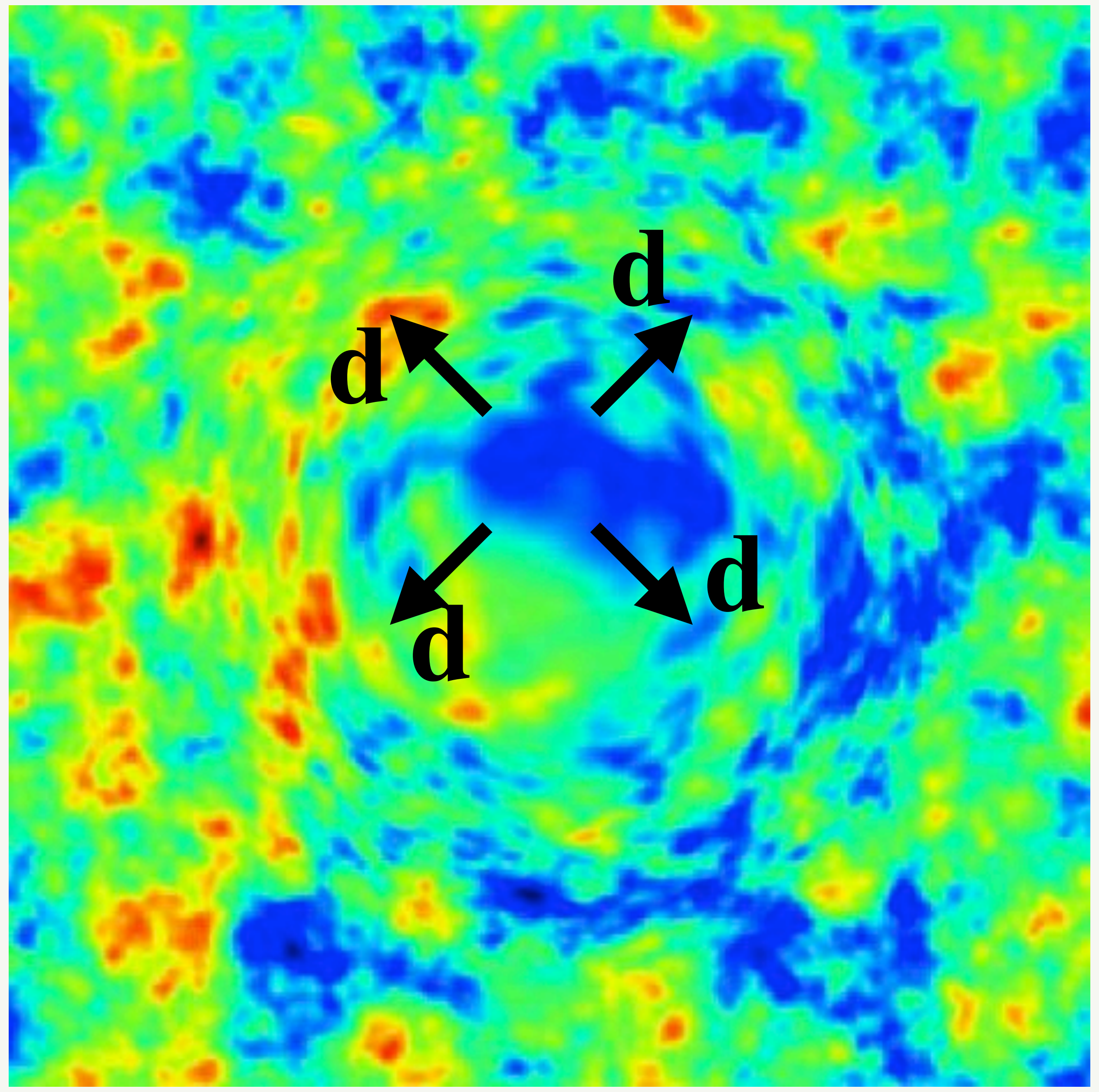
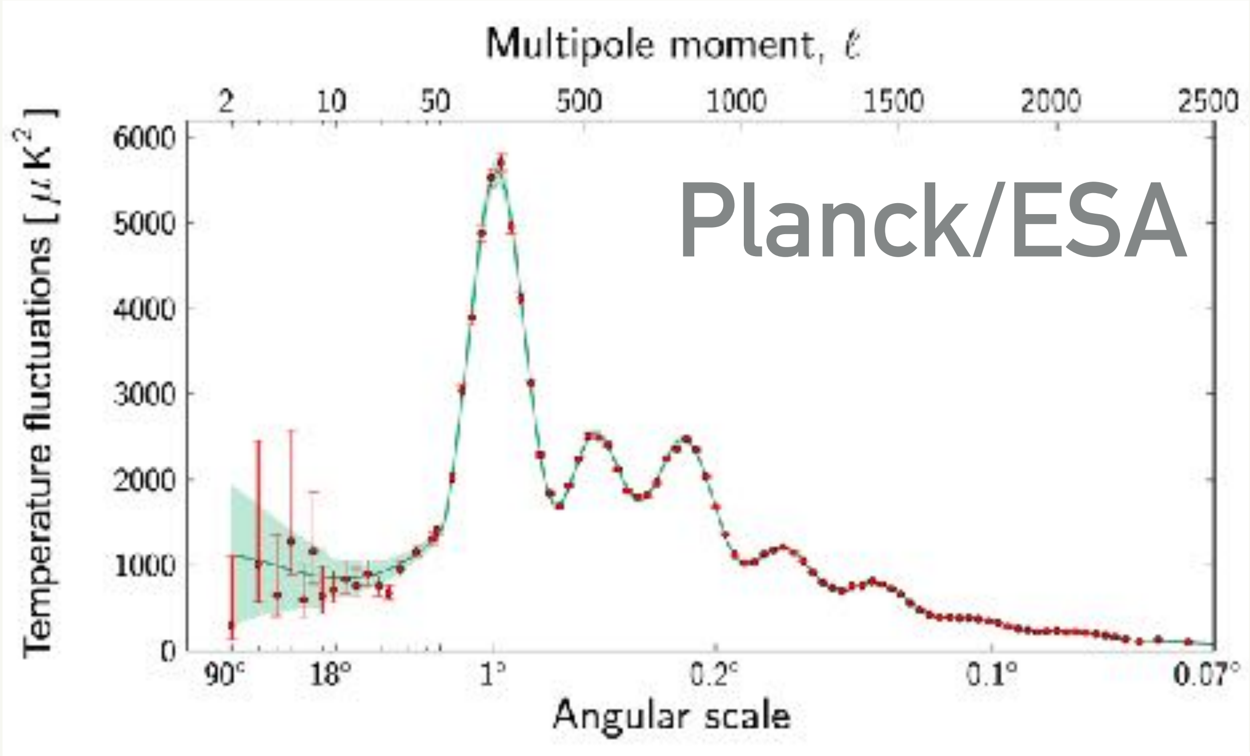
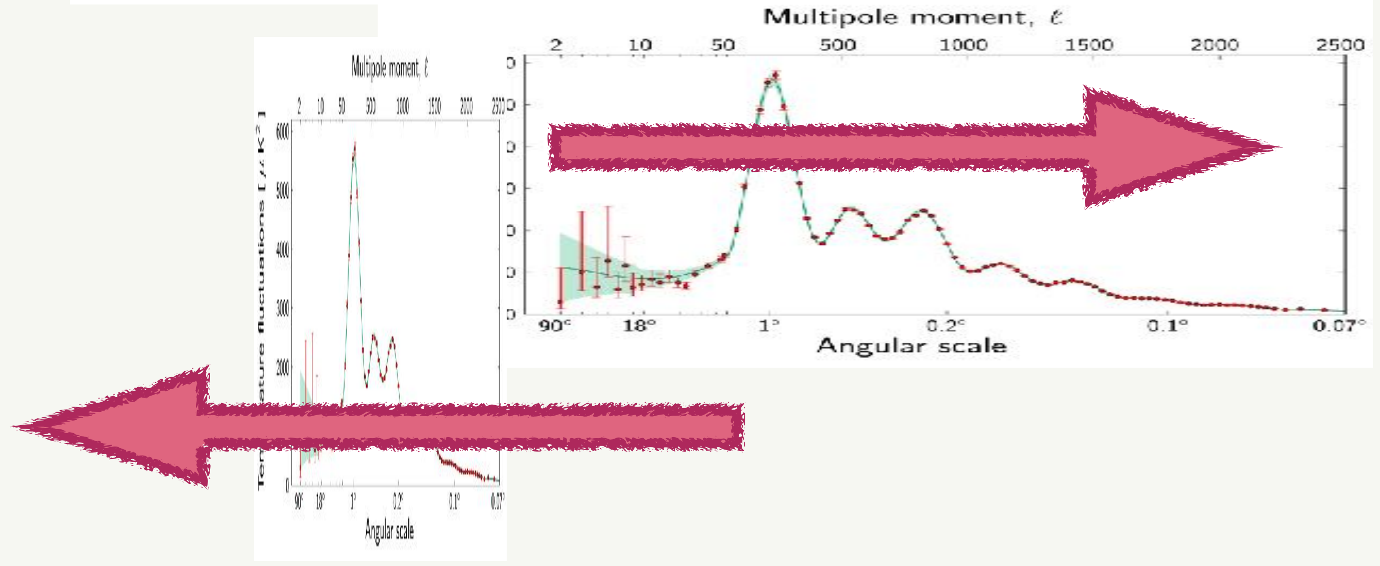
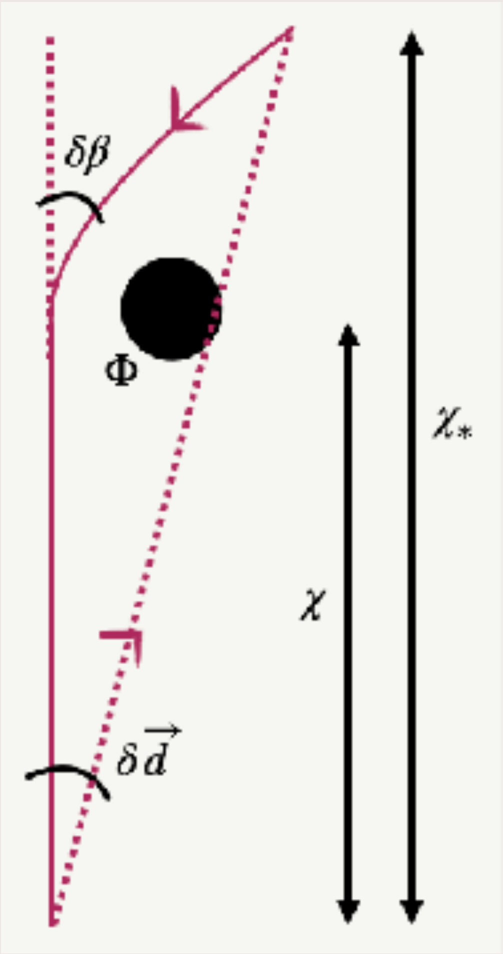


Lensing effects on the CMB power spectra

1. Large-scale lenses smooth the peaks of the observed power spectrum
2. Small-scale lenses introduce small-scale power into the damping tail







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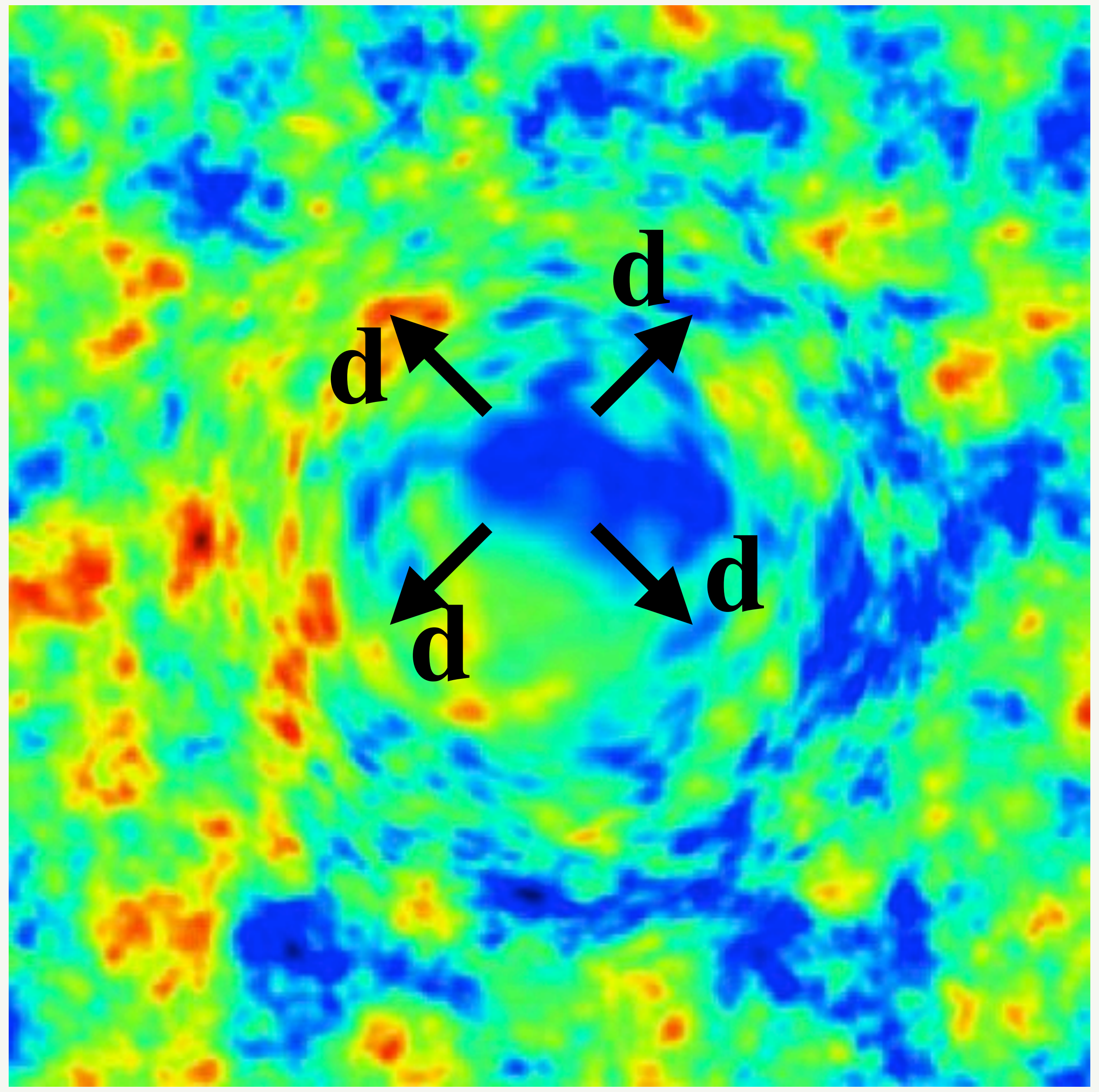
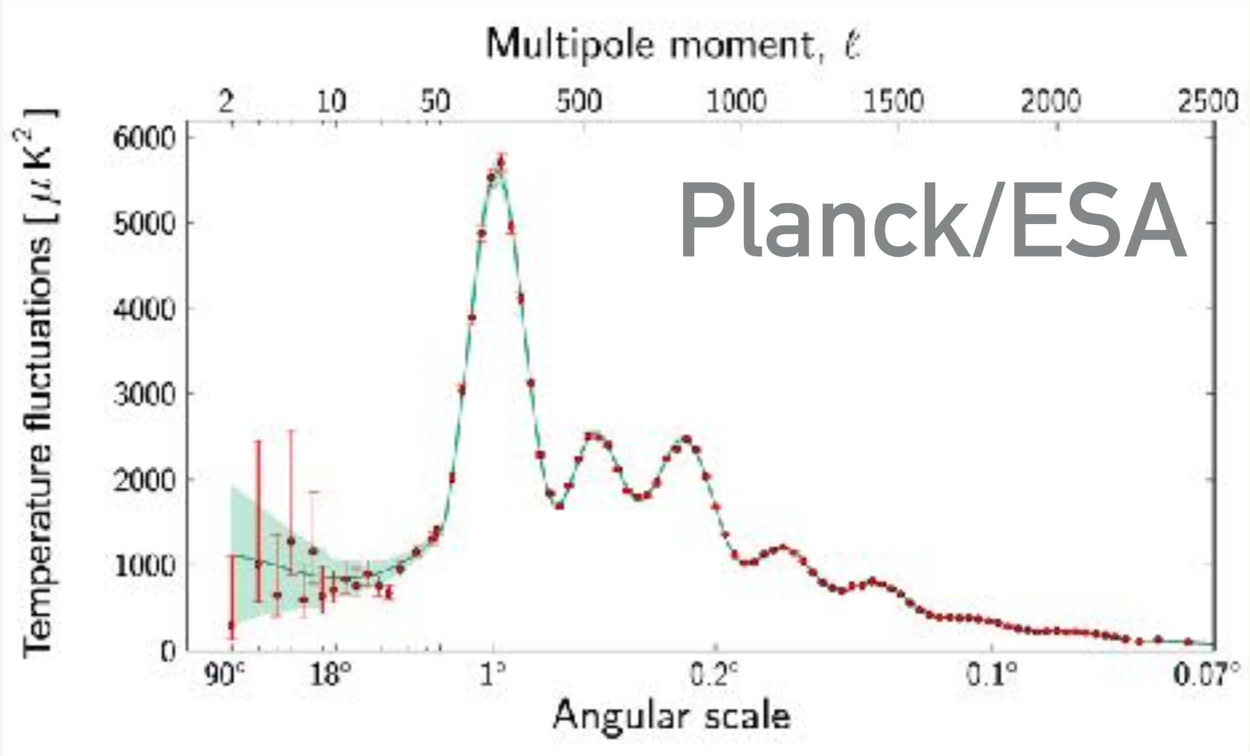
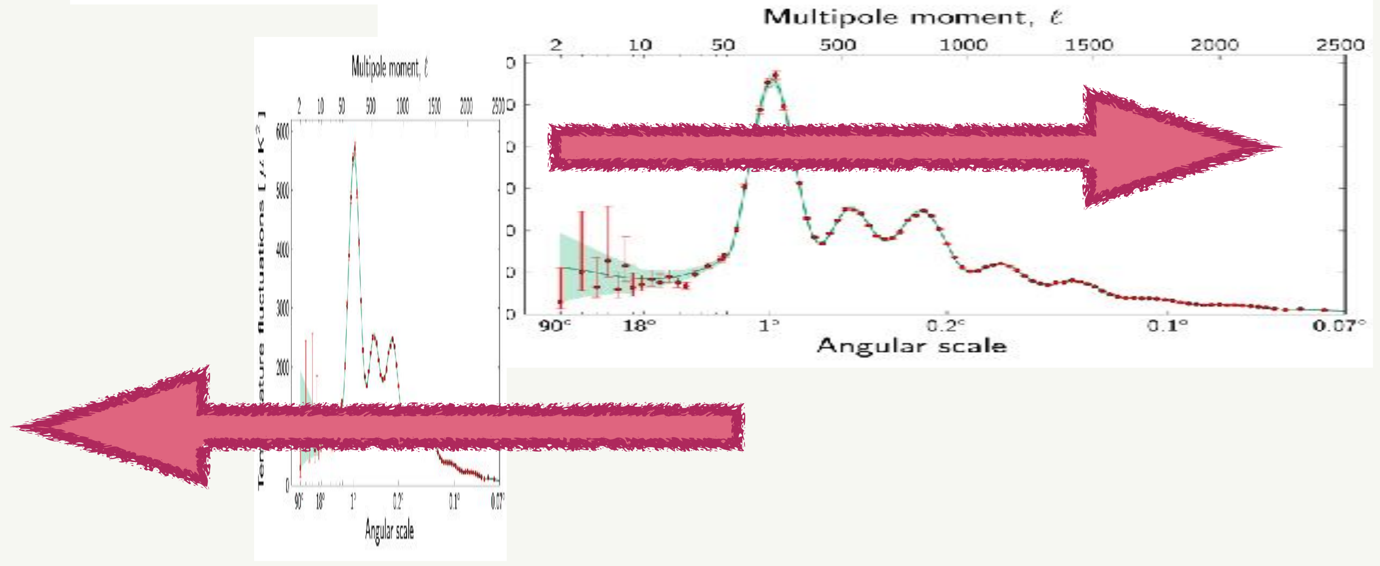


Lensing effects on the CMB power spectra

1. Large-scale lenses smooth the peaks of the observed power spectrum
2. Small-scale lenses introduce small-scale power into the damping tail
3. Lensing of the CMB polarisation causes power to leak from E- to B-modes (c.f. lensed B-modes are nearly white, $C_\ell^{BB} \sim 2 \times 10^{-6} \mu\text{K}^2$ corresponding to instrumental white-noise level of 5μK-arcmin)





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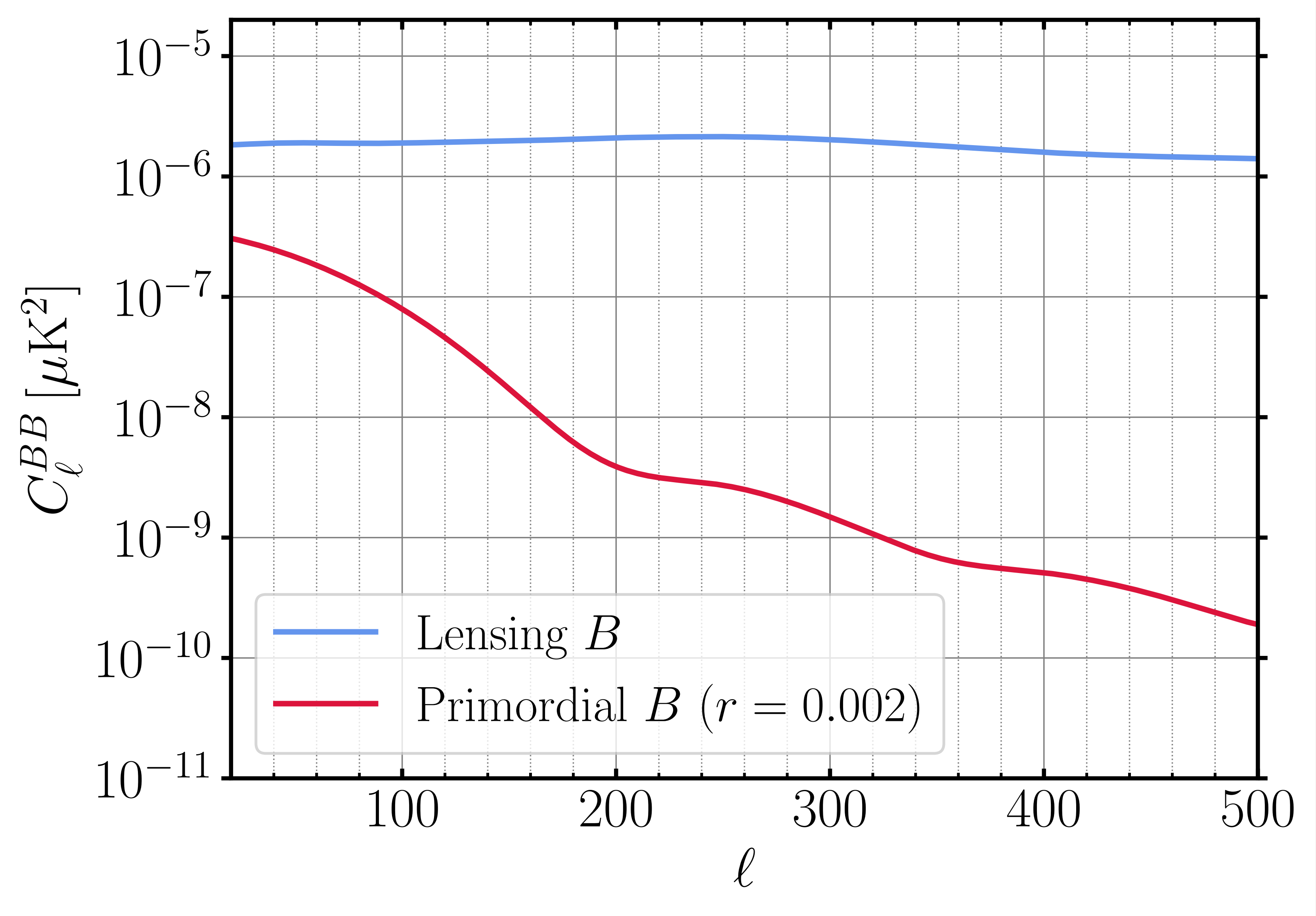
B-modes from the early Universe and delensing

“Answering the Big Questions”



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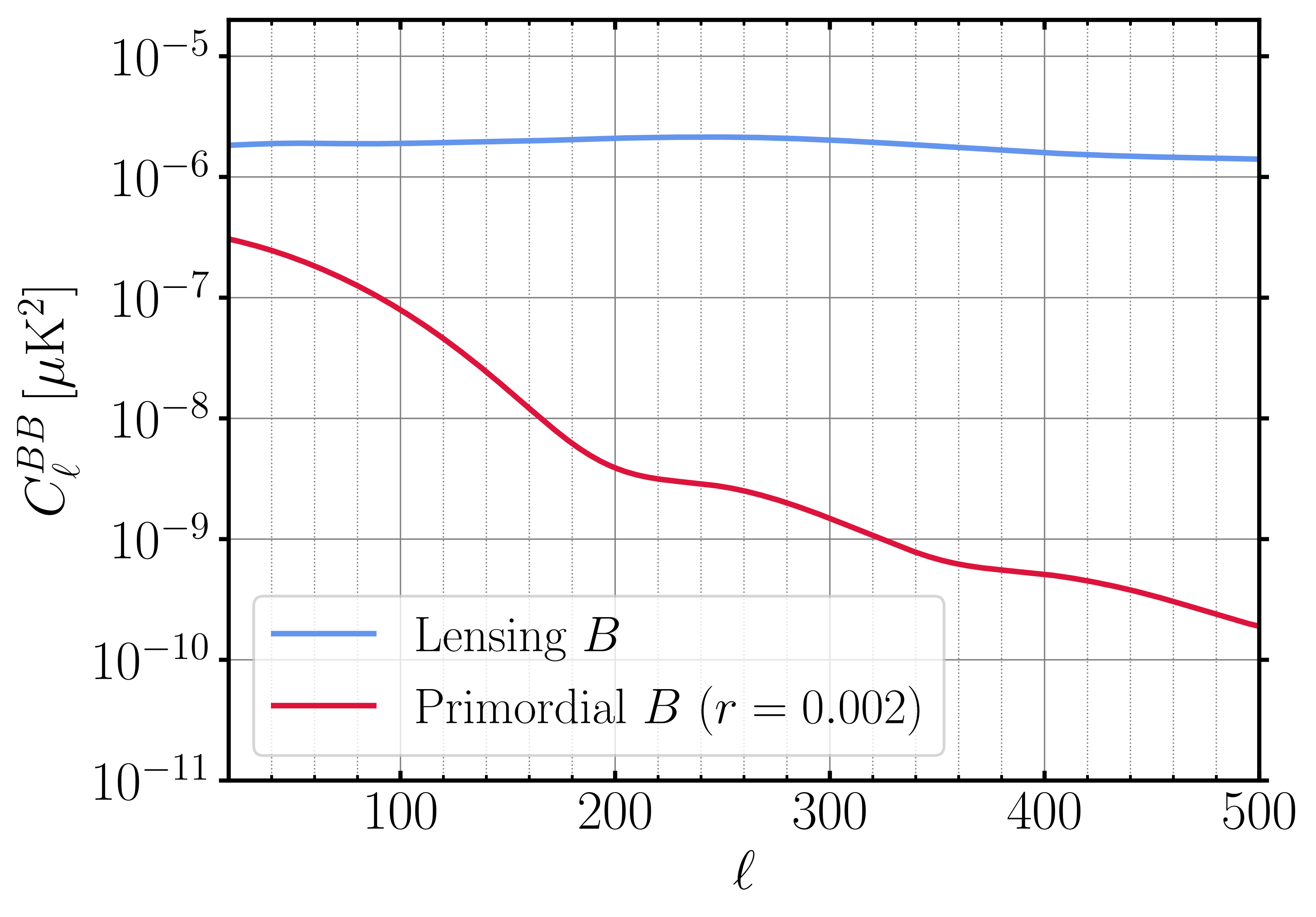

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$$\widetilde{T}(\mathbf{l}) = T(\mathbf{l}) - \int \frac{\mathrm{d}^2\mathbf{l}'}{(2\pi)^2} \left[\mathbf{l}'\ \mathbf{T}(\mathbf{l}')\right] \cdot (\mathbf{l} - \mathbf{l}')\, \phi\,(\mathbf{l} - \mathbf{l}')$$

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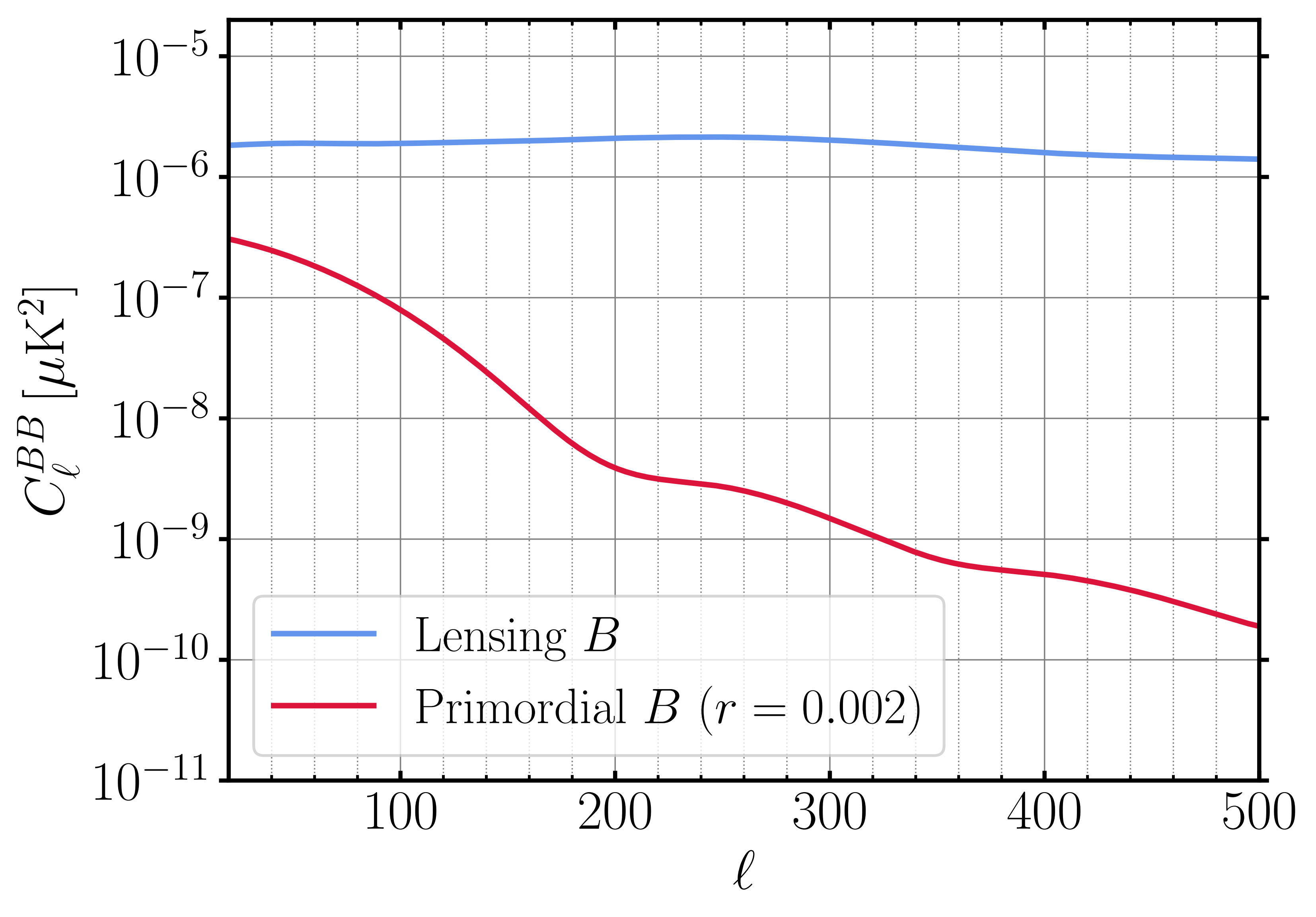

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- Similarly, we obtain lensing B-modes from the convolution of E-modes with the lensing potential:

$$\tilde{B}(\mathbf{l}) = \int \frac{\mathrm{d}^2\mathbf{l}'}{(2\pi)^2} W(\mathbf{l}, \mathbf{l}')\, E(\mathbf{l}')\, \phi\,(\mathbf{l} - \mathbf{l}')$$

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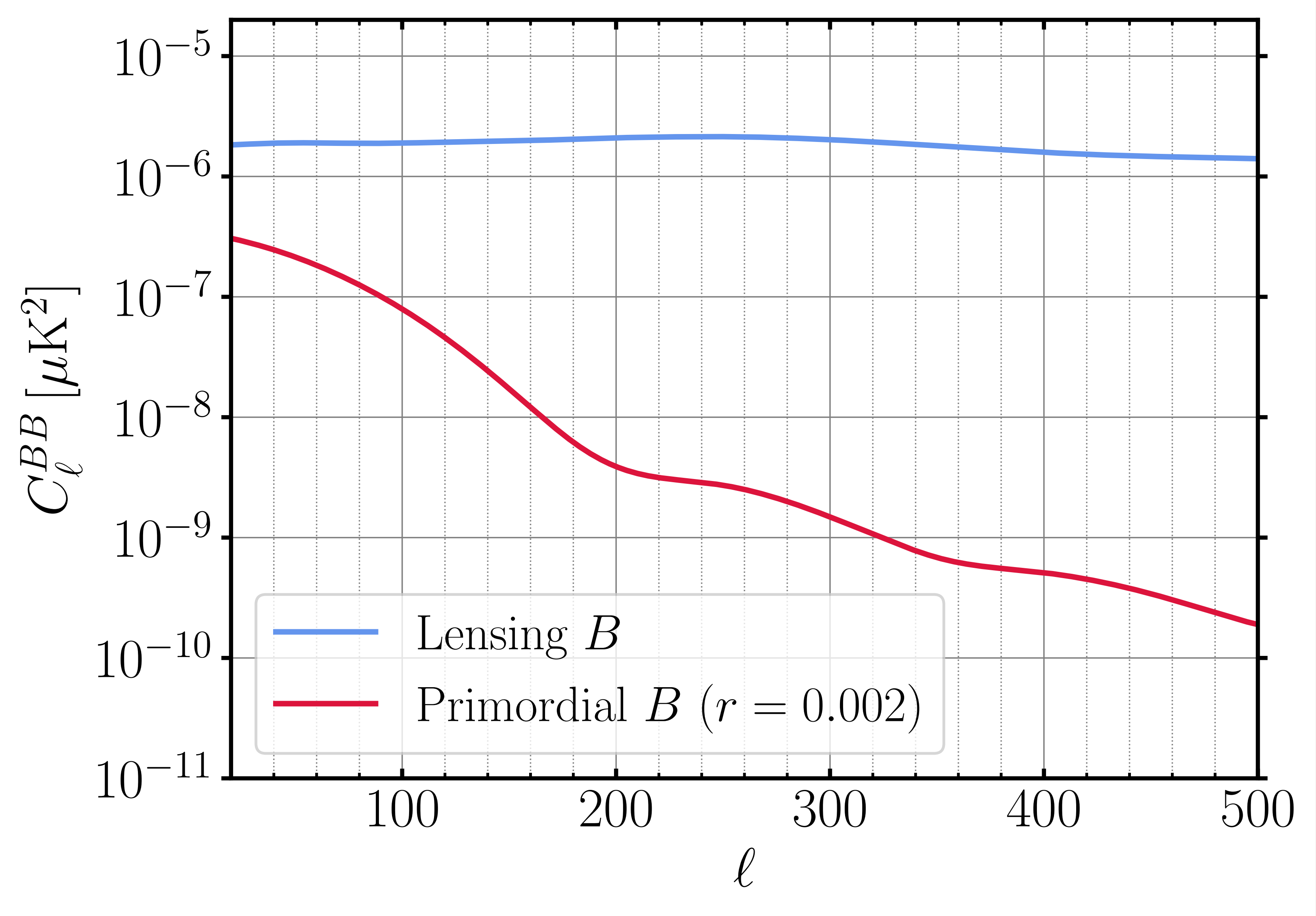

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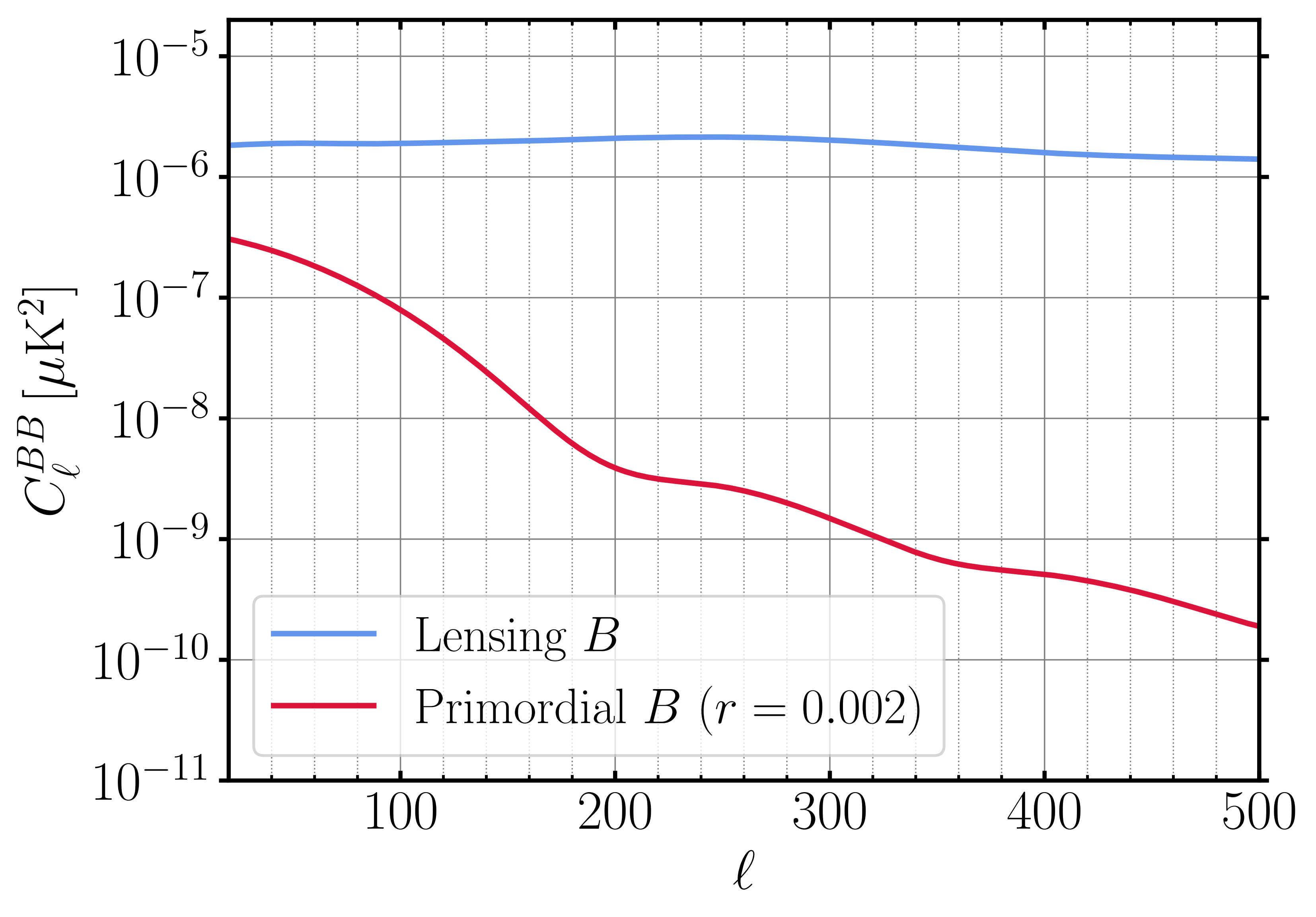

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- Want $f(\mathbf{l},\mathbf{l}')$ to minimise residual B-mode power, and then:

$$C_l^{BB,\mathrm{res}} = \int \frac{\mathrm{d}^2\mathbf{l}'}{(2\pi)^2} W^2(\mathbf{l},\mathbf{l}')\,C_{l'}^{EE}\,C_{|\mathbf{l}-\mathbf{l}'|}^{\kappa\kappa}\left(1-\rho^2_{|\mathbf{l}-\mathbf{l}'|}\right)$$

Correlation coefficient between the true lensing signal and the tracer I

$$\rho = \frac{C_l^{\kappa I}}{\sqrt{C_l^{\kappa\kappa}C_l^{II}}}$$

How to measure CMB lensing?

CMB lensing reconstruction

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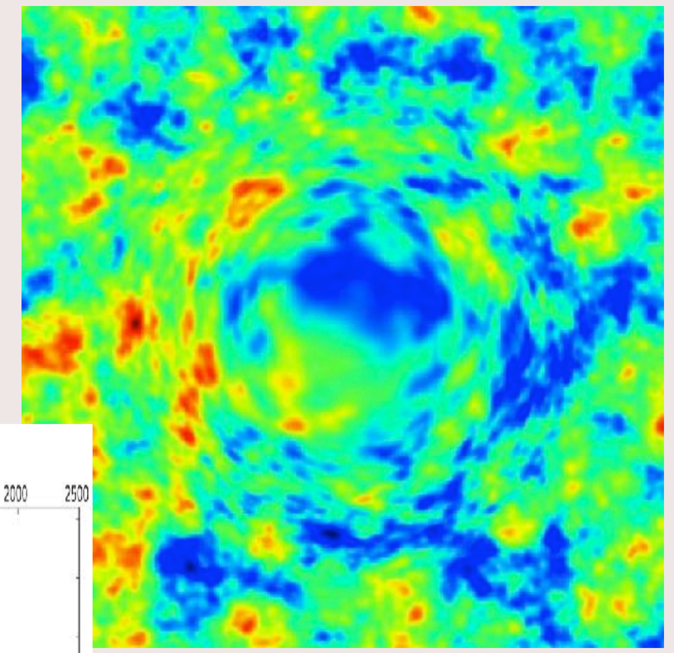
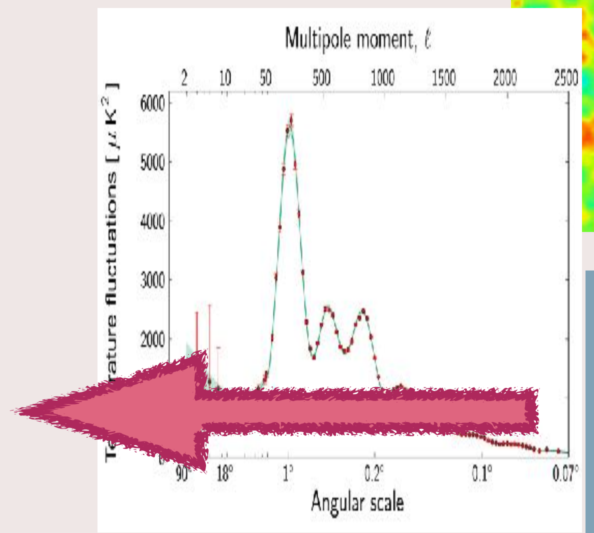
Statistical isotropy on CMB maps: $\langle T(\mathbf{l})\, T(\mathbf{l}')\rangle = (2\pi)^2\, C_l\, \delta^{(D)}(\mathbf{l}-\mathbf{l}')$

How to measure CMB lensing?



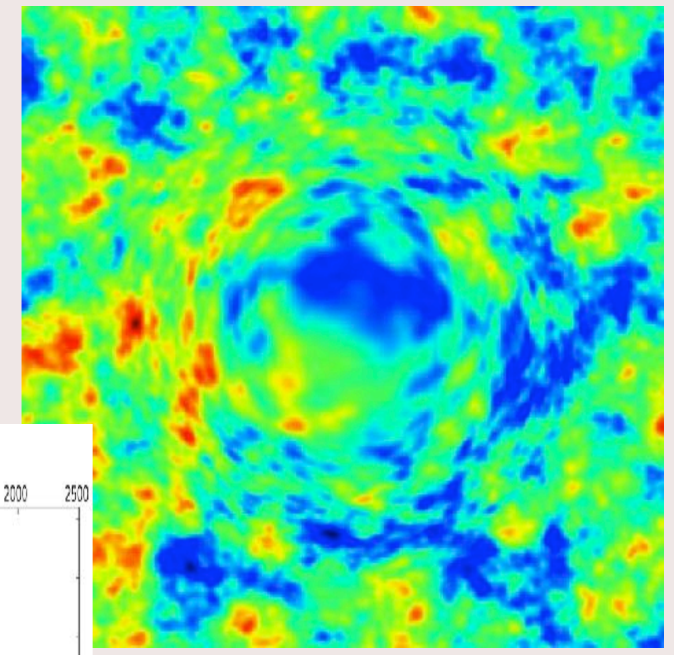
CMB lensing reconstruction

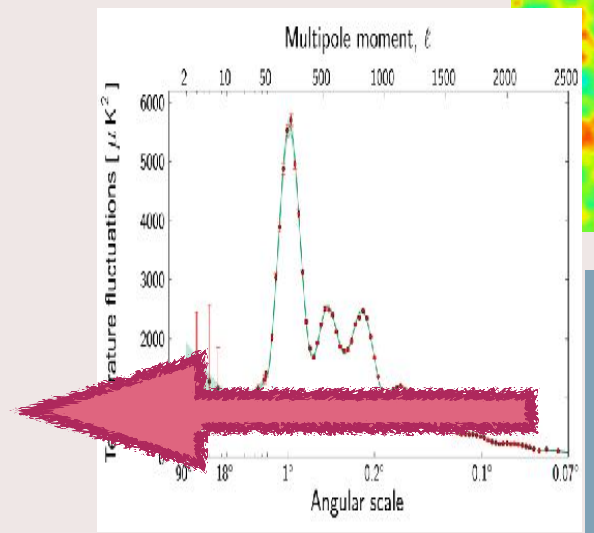


shift to larger angular scales

Statistical isotropy on CMB maps: $\langle T(\mathbf{l})\, T(\mathbf{l}') \rangle = (2\pi)^2\, C_l\, \delta^{(D)}(\mathbf{l} - \mathbf{l}')$

How to measure CMB lensing?





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CMB lensing breaks translation invariance, introducing new correlations:

$$\langle \widetilde{T}(\mathbf{l})\, \widetilde{T}(\mathbf{l}')\rangle_{\mathrm{CMB}} = \phi(\mathbf{L}) \left[\tilde{C}_l^{TT}(\mathbf{L}\cdot\mathbf{l}) + \tilde{C}_{l'}^{TT}(\mathbf{L}\cdot\mathbf{l}')\right] \equiv K(\mathbf{l},\mathbf{L})\,\phi(\mathbf{L})$$

Recall:

$$\widetilde{T}(\mathbf{l}) = T(\mathbf{l}) - \int \frac{\mathrm{d}\mathbf{l}'}{(2\pi)^2} \left[\mathbf{l}'\, \mathbf{T}(\mathbf{l}')\right]\cdot(\mathbf{l}-\mathbf{l}')\,\phi(\mathbf{l}-\mathbf{l}')$$

with $\mathbf{L} = \mathbf{l}' - \mathbf{l}$.

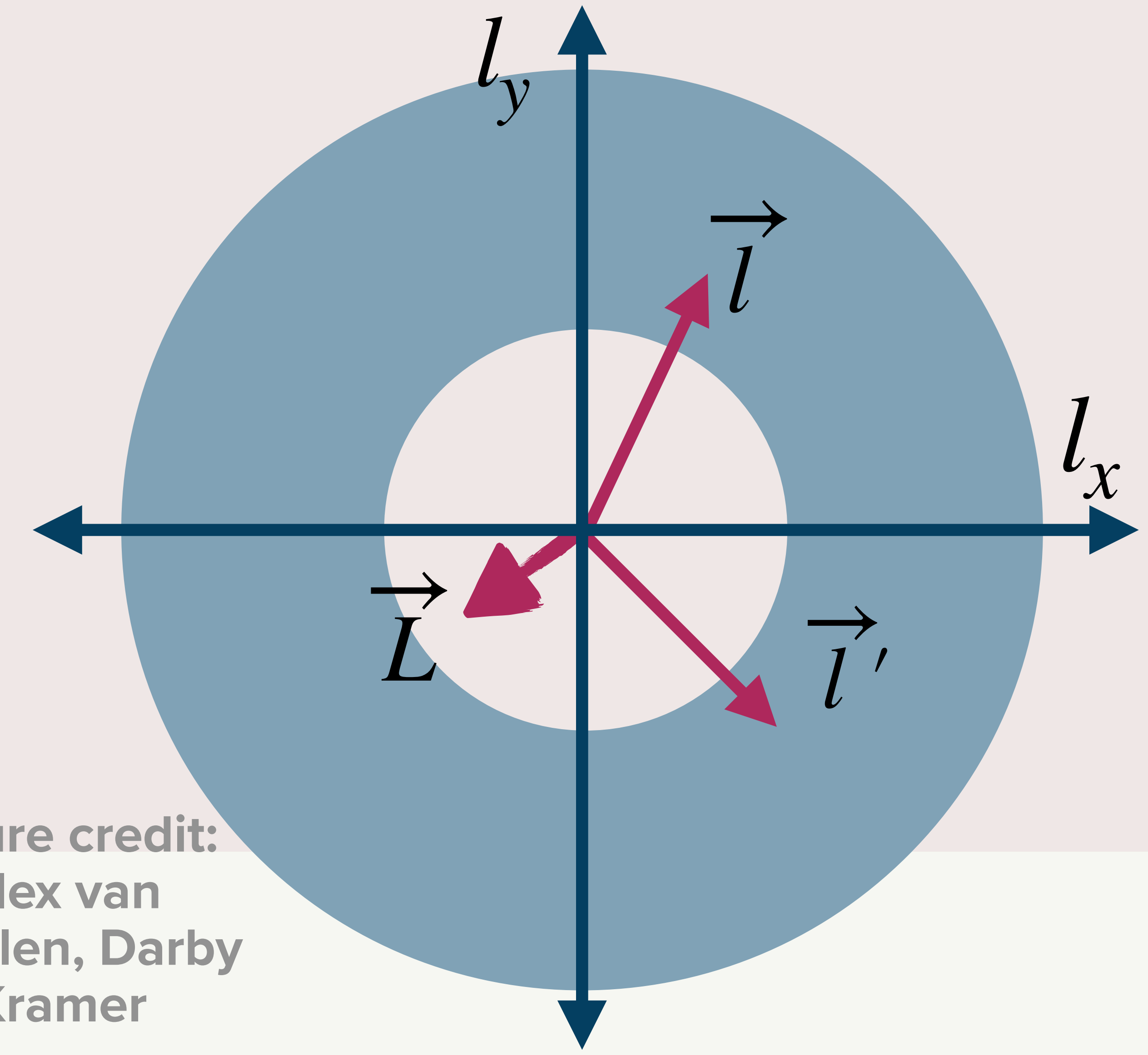

Figure credit: Alex van Engelen, Darby Kramer

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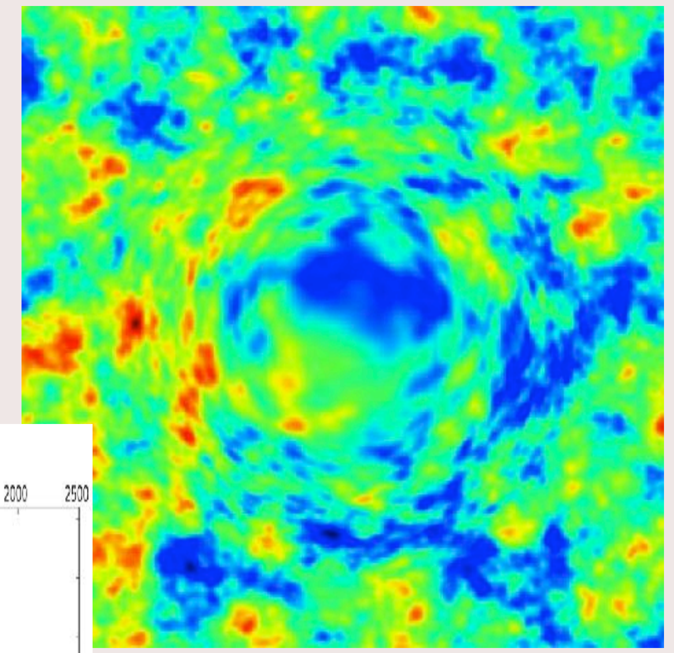
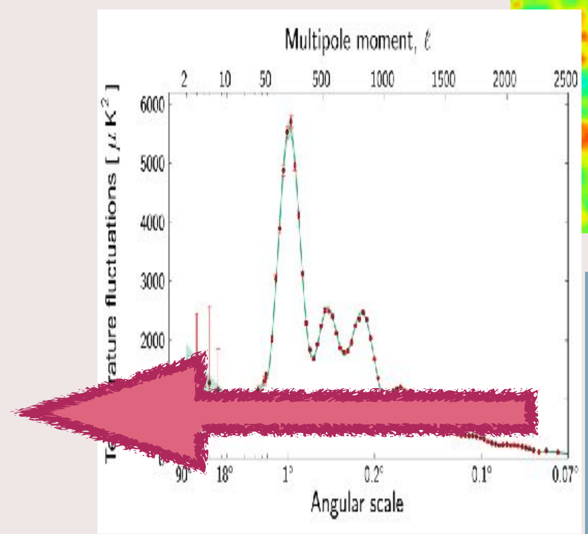
How to measure CMB lensing?





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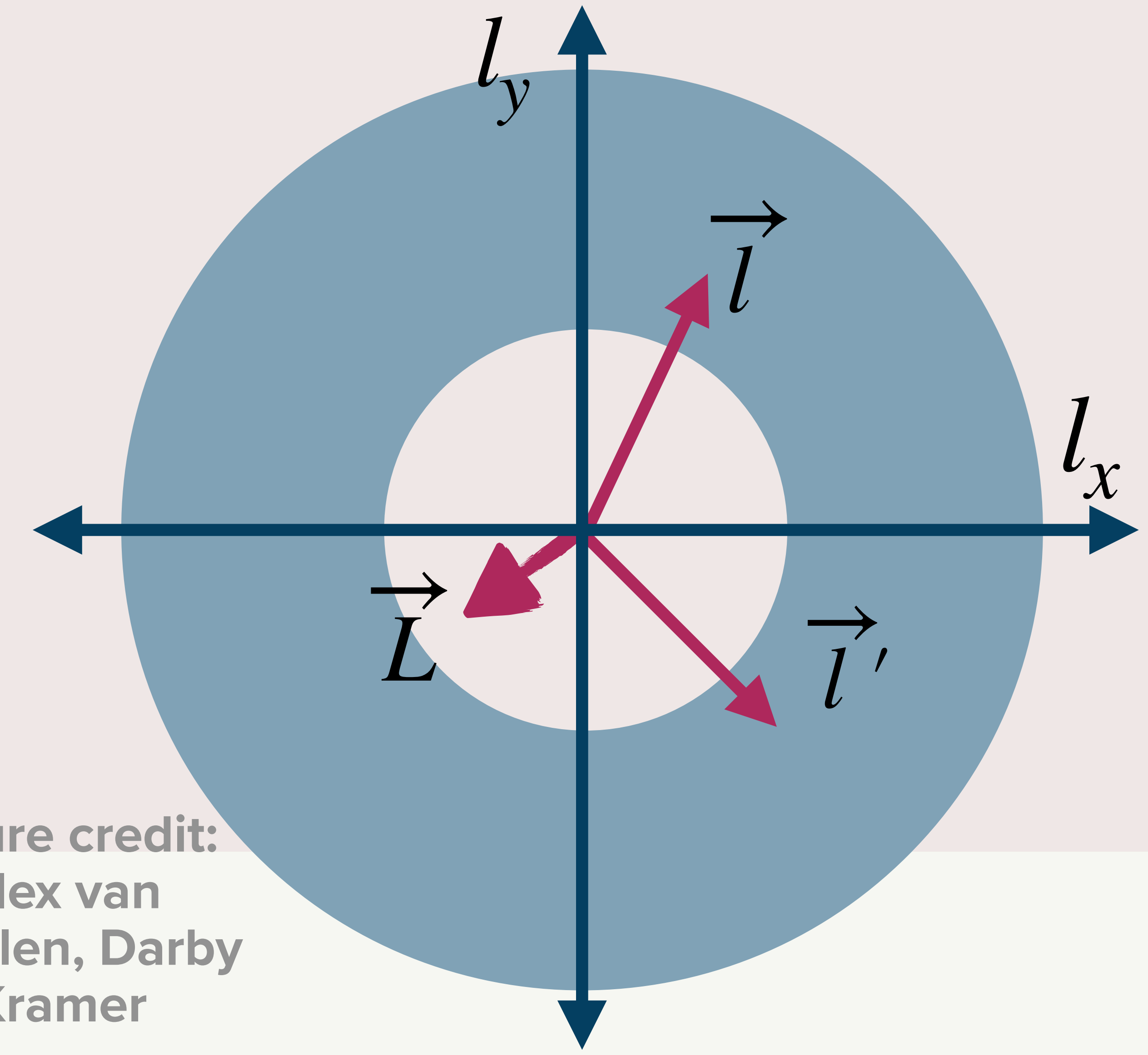
with $\mathbf{L} = \mathbf{l}' - \mathbf{l}$. Motivates deriving lensing estimator of the form



$$\boxed{\hat{\phi} \sim \frac{\widetilde{T}(\mathbf{l})\, \widetilde{T}(\mathbf{L}-\mathbf{l})}{K(\mathbf{l},\mathbf{L})}}$$

Combine all pairs with inverse-variance weighting:

$$\hat{\phi} \sim \int \frac{\mathrm{d}^2\mathbf{l}}{(2\pi)^2} \frac{1}{\mathrm{var}\left[\tilde{\mathrm{T}}(\mathbf{l})\, \tilde{\mathrm{T}}(\mathbf{L}-\mathbf{l})/\mathrm{K}(\mathbf{l},\mathbf{L})\right]} \frac{\widetilde{T}(\mathbf{l})\, \widetilde{T}(\mathbf{L}-\mathbf{l})}{K(\mathbf{l},\mathbf{L})}$$

Figure credit: Alex van Engelen, Darby Kramer

How to measure CMB lensing ?

CMB lensing reconstruction

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CMB lensing reconstruction

Arrive to expression for quadratic estimator:

$$\hat{\phi}(\mathbf{L}) \sim \int \frac{\mathrm{d}^2\mathbf{l}}{(2\pi)^2} \left[\frac{(\mathbf{L}-\mathbf{l})\cdot\mathbf{L}\, C_{|\mathbf{L}-\mathbf{l}|} + \mathbf{l}\cdot\mathbf{L}\, C_l}{2\, C_l^{\mathrm{tot}}\, C_{|\mathbf{L}-\mathbf{l}|}^{\mathrm{tot}}} \right] \cdot\ \widetilde{T}(\mathbf{l})\widetilde{T}(\mathbf{L}-\mathbf{l})$$

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How to measure CMB lensing $\underbrace{}_{\text{WITH CMB POLARIZATION FIELD}}$?

CMB lensing reconstruction

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In general, for two CMB fields X, Y, the estimators are of the form:

$$\hat{\phi}_{XY}(\mathbf{L}) = N_{XY}(\mathbf{L}) \int \frac{d^2\mathbf{l}}{(2\pi)^2}\, \tilde{X}(\mathbf{l})\, \tilde{Y}^*(\mathbf{l}-\mathbf{L})\; g_{XY}(\mathbf{l}, \mathbf{L})$$

How to measure CMB lensing $\underbrace{}_{\text{WITH CMB POLARIZATION FIELD}}$?

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For the case $X = E, Y = B$, the minimum variance estimator has $g_{EB}(\mathbf{l},\mathbf{L}) = \sin(2\varphi)\, \dfrac{\mathbf{l} \cdot \mathbf{L}\, C_l^{EE} + (\mathbf{L}-\mathbf{l}) \cdot \mathbf{L}\, C_{|\mathbf{L}-\mathbf{l}|}^{BB}}{\tilde{C}_l^{E,\text{tot}}\, \tilde{C}_{|\mathbf{L}-\mathbf{l}|}^{B,\text{tot}}}$

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How to measure CMB lensing ?

WITH CMB POLARIZATION FIELD

CMB lensing reconstruction

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For references, see e.g. Lewis & Challinor

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How to measure CMB lensing ?

WITH CMB POLARIZATION FIELD

CMB lensing reconstruction

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The integral is a harmonic-space convolution $\Longrightarrow$ Write as real-space product

$$\hat{\phi}(\mathbf{L}) = -N(\mathbf{L}) \int d^2\mathbf{x}\; e^{-i\mathbf{L}\cdot\mathbf{x}}\; \nabla \cdot \left[F_1(\mathbf{x})\, \nabla F_2(\mathbf{x}) \right]$$

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The CMB lensing power spectrum

Not as straightforward as one would imagine

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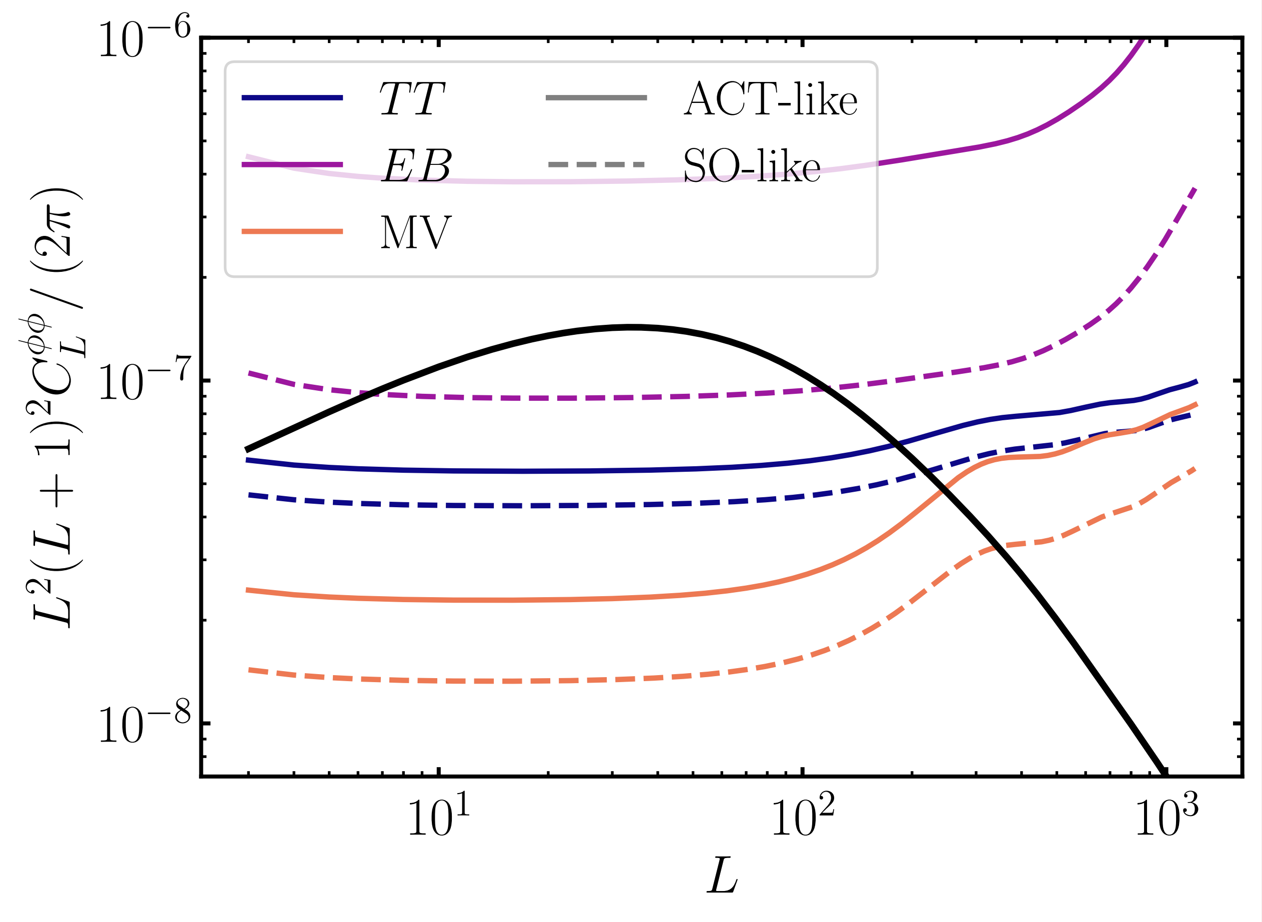
What is $N(\mathbf{L})$?



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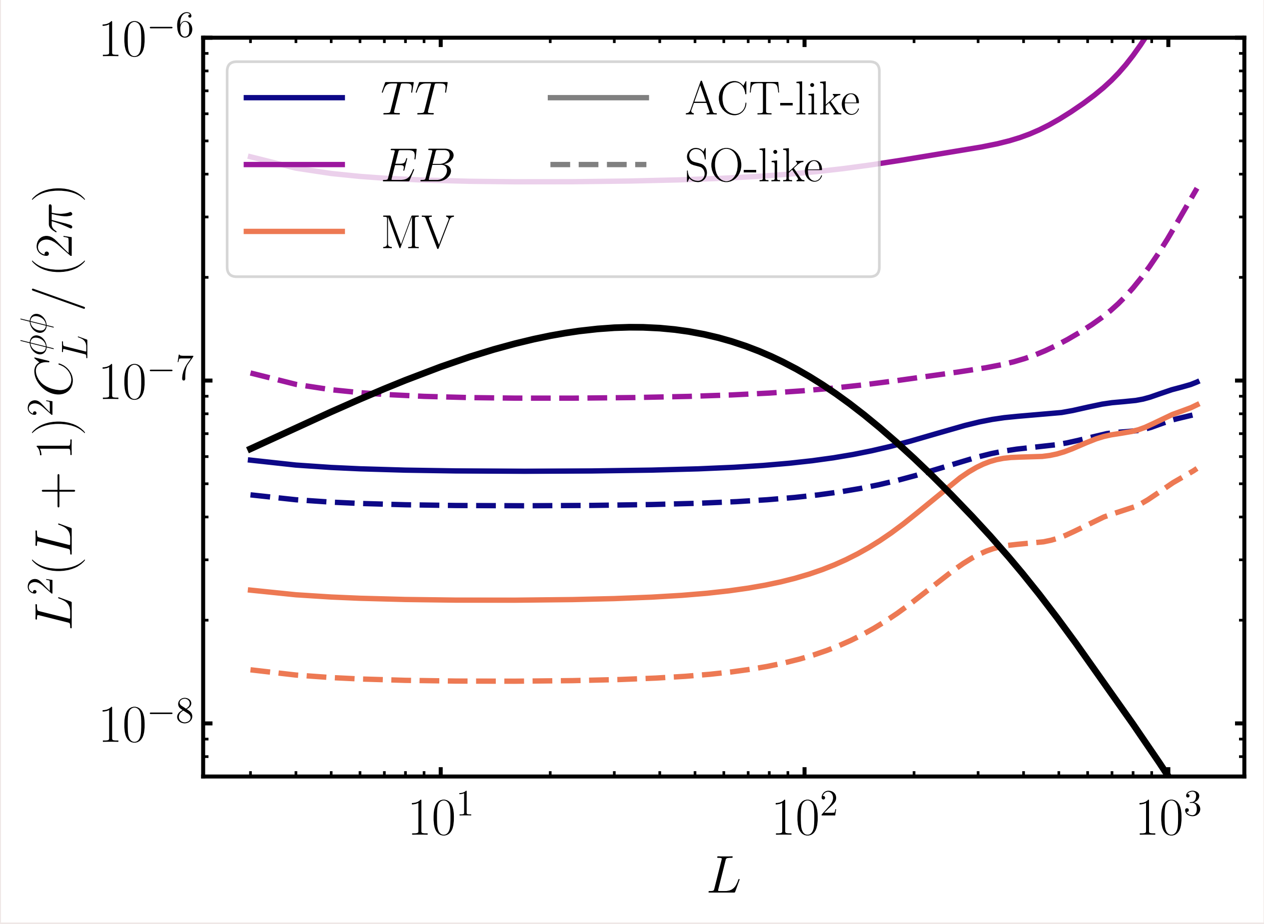
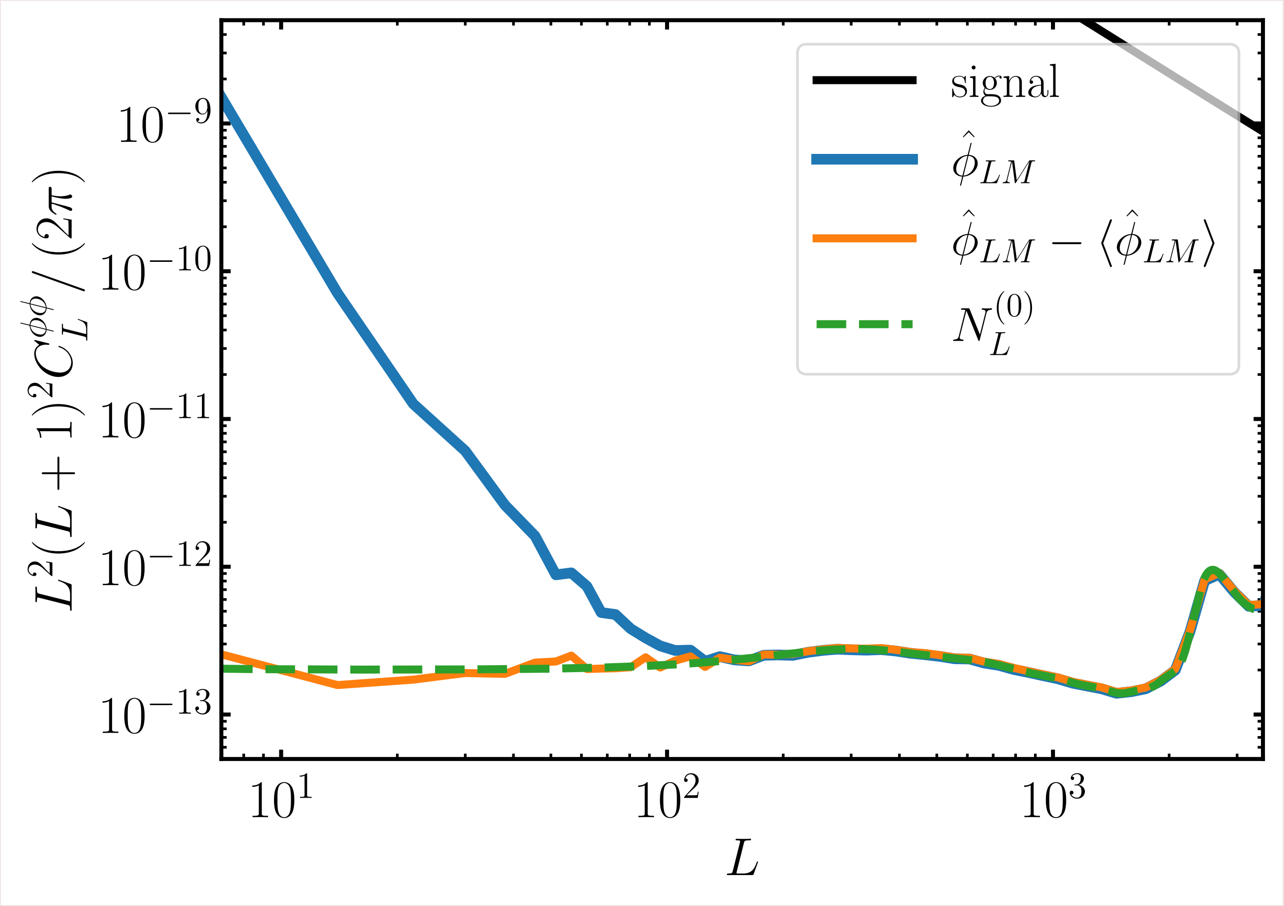


How come I pick up a signal when I try to measure lensing on the unlensed CMB?

What happens in the presence of a mask?



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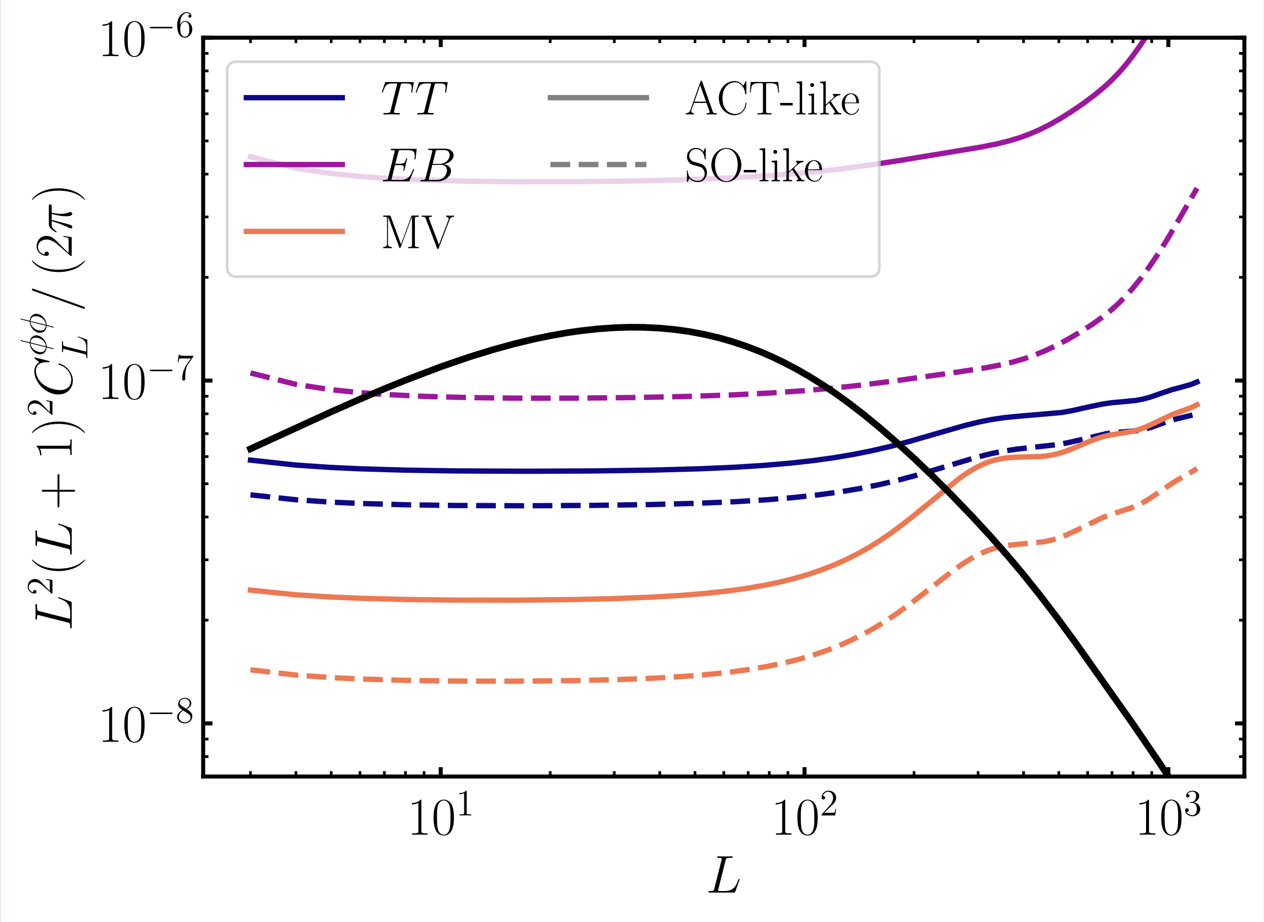

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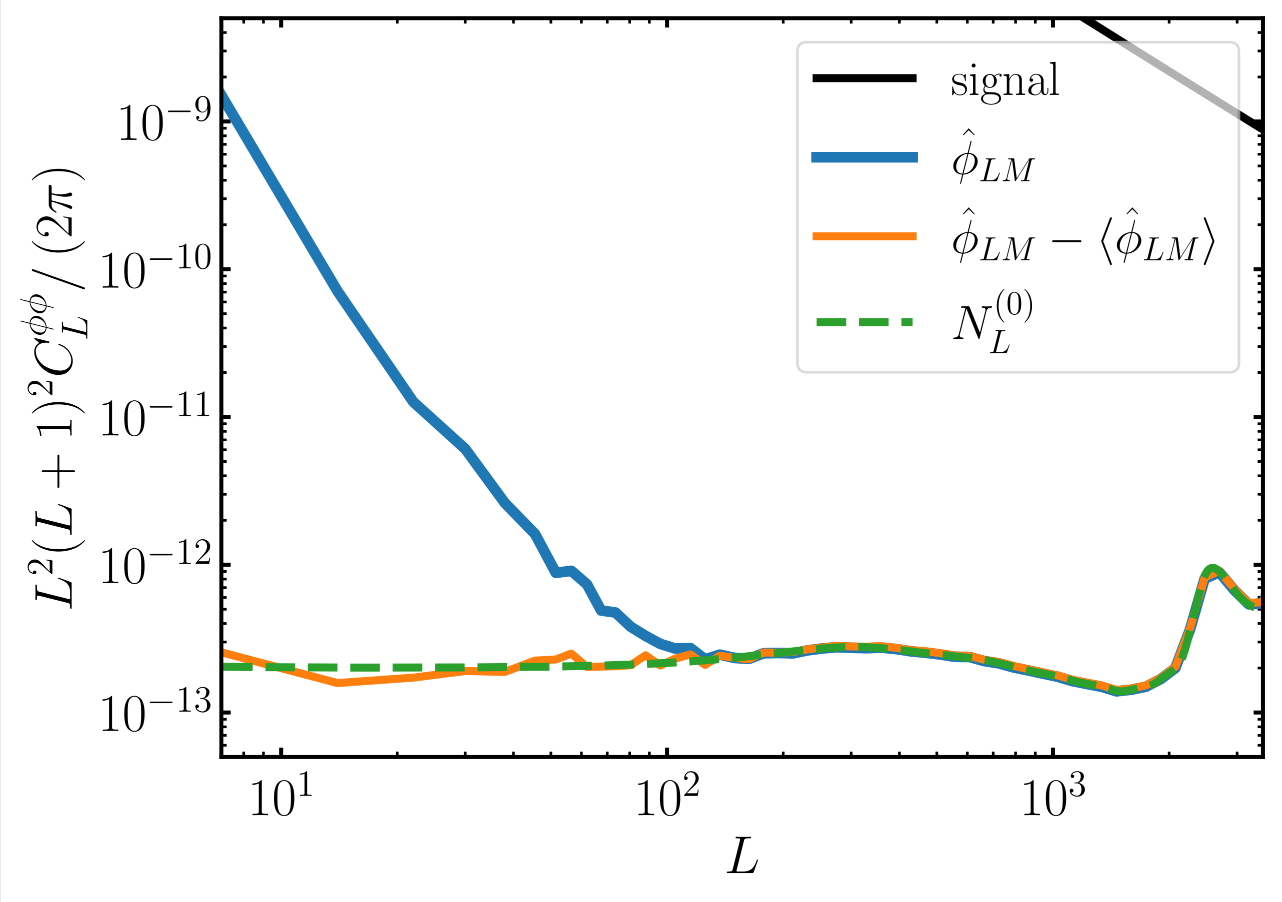
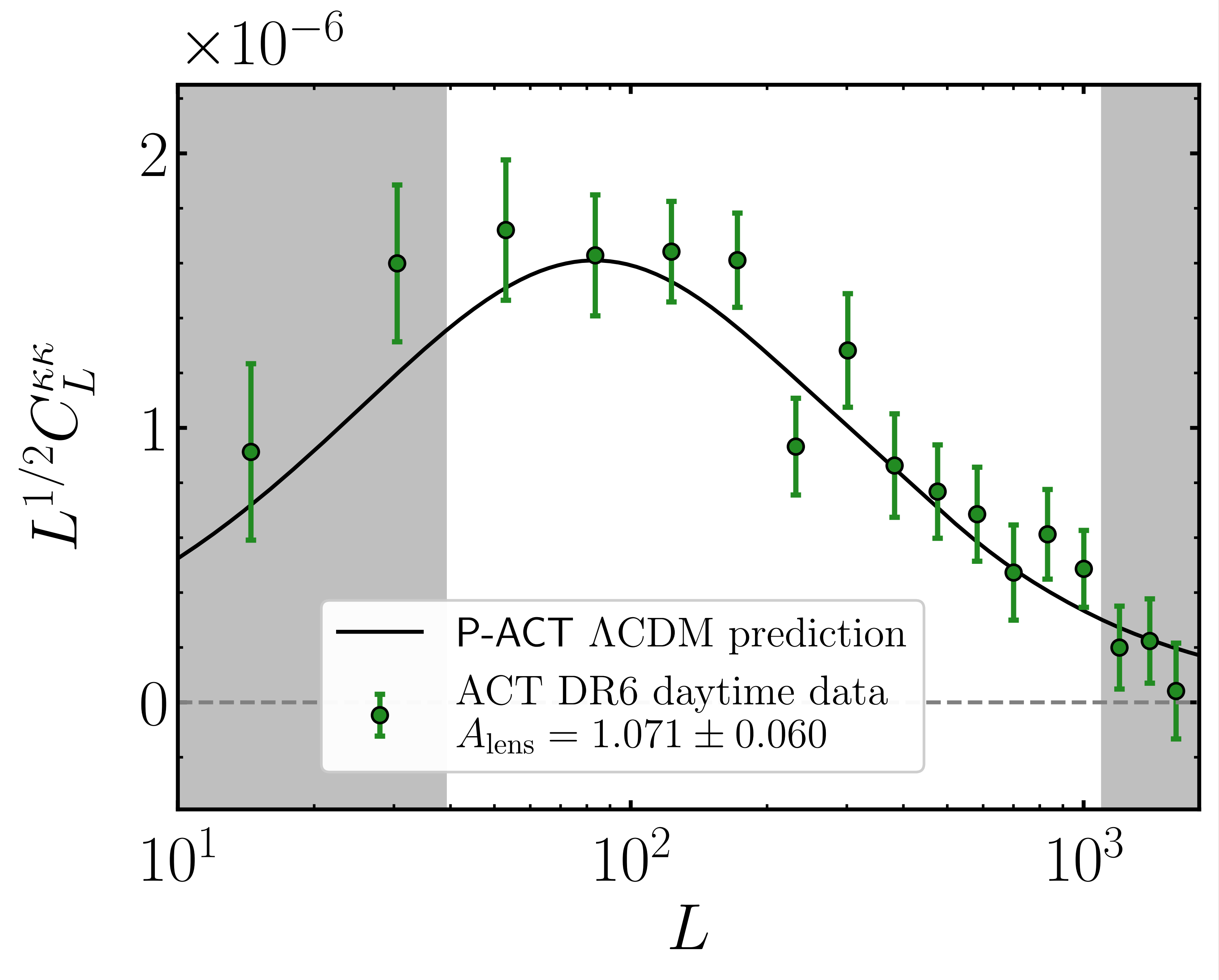


Lensing amplitude with respect to ΛCDM prediction $\left(A_{\rm lens}\right)$?



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